

Vizsgálati eredményekre fektetett
REGRESSZIÓS EGYENLETEK
függvényállandóinak meghatározása
a legkisebb hibanégyzetösszegek módszerével

(Elnézést a kézírásért...)

TARTALOM

- 1) Determinánsok kiszámítása
- 2) $y = a \times x$
- 3) $y = a \times x + b$
- 4) $y = a \times x^2 + b \times x$
- 5) $y = a \times x^2 + b \times x + c$
- 6) $y = a \times x^2 + b \times x + c$, ha a parabola három adott pontra fekszik
- 7) $y = a \times x^3 + b \times x^2 + c \times x$
- 8) $y = a \times x^{\frac{1}{3}} + b \times x^{\frac{1}{2}} + c \times x$
- 9) $y = a \times x^b$
- 10) $y = a \times e^{b \times x}$
- 11) $y = \frac{1}{a \times x}$
- 12) $y = \frac{1}{a \times x + 1}$
- 13) $y = \frac{1}{a \times x + b}$
- 14) $y = \frac{a \times x + b}{x + 1}$
- 15) $y = \frac{a \times x}{x + a - 1}$

$$16) \quad z = a \times y + b \times x + c$$

$$17) \quad z = a \times x^2 + b \times x + y + c$$

$$18) \quad z = a \times x^2 + b \times x + c \times y + d$$

$$19) \quad z = a \times x^2 + b \times y^2 + c \times x + d \times y + e$$

$$20) \quad z = \frac{a \times x + b \times y + c}{x + 1}$$

$$21) \quad z = \frac{a \times x + b \times y - x \times y + c}{x + 1}$$

$$22) \quad z = \frac{a \times x^2}{y - 1} + \frac{b \times x}{y - 1}$$

$$23) \quad z = \frac{a \times x^2}{y - 1} + \frac{b \times x}{y - 1} + c$$

$$24) \quad z = \frac{a \times (1 - x^2)}{y} + \frac{b \times (1 - x)}{y} + c$$

$$25) \quad z = \frac{x}{a \times \ln y} + \frac{a}{b}$$

$$26) \quad z = \frac{\ln y \pm b \times x}{a \times x}$$

27) Korrelációs jellemzők

1) Determinánsok kiszámítása

$$D = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$D_a = \begin{vmatrix} g & b & c \\ h & d & e \\ i & e & f \end{vmatrix}$$

$$D_b = \begin{vmatrix} a & g & c \\ b & h & e \\ c & i & f \end{vmatrix}$$

$$D_c = \begin{vmatrix} a & b & g \\ b & d & h \\ c & e & i \end{vmatrix}$$

$$D = +a \underbrace{(af - c^2)}_{D_{01}} - b \underbrace{(bf - ec)}_{D_{02}} + c \underbrace{(be - cd)}_{D_{03}}$$

$$D_a = +g \cdot D_{01} - b \underbrace{(hf - ei)}_{D_{a2}} + c \underbrace{(hc - di)}_{D_{a3}}$$

$$D_b = +a \cdot D_{a2} - g \cdot D_{02} + c \underbrace{(bi - ch)}_{D_{b3}}$$

$$D_c = +a \cdot (-D_{a3}) - b \cdot D_{b3} + g \cdot D_{03}$$

Sarrus - szabály a harmadrendű determinánsok kiszámítására

A determinánsok negyedik és ötödik oslopokat hárva át a harmadrendű determinánsokat számítjuk ki. A sarrus-szabály szerint a harmadrendű determináns címlapjának, azaz a függőleges sorban álló elemek sorrendjéből levonjuk a mellekeltlőkben álló elemek sorrendjének összegét.

A Sarrus-szabály szerint a D determináns címlapját meghagyjuk, ha a függőleges sorban álló elemek sorrendjük összegéből levonjuk a mellekeltlőkben álló elemek sorrendjük összegét.

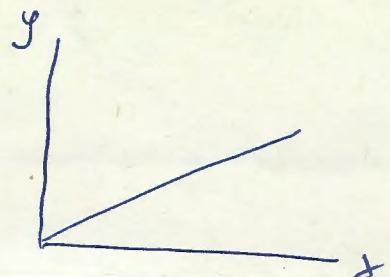
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \begin{array}{c} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{array}$$

$$D = a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{13} \cdot a_{21} \cdot a_{32} - a_{31} \cdot a_{22} \cdot a_{13} - a_{32} \cdot a_{23} \cdot a_{11} - a_{33} \cdot a_{21} \cdot a_{12}$$

Oripon átmens" egyenes.

2) $y = ax$

$$y = ax$$



$$F = \sum (y - ax)^2$$

$$\frac{dF}{da} = -2 \sum (y - ax)x = 0$$

$$\sum xy - a \sum x^2 = 0$$

$$\sum xy = a \sum x^2$$

$$a = \frac{\sum xy}{\sum x^2}$$

$$\text{Meghatározandó} = \bar{x} \cdot \bar{y} \Leftrightarrow \sum x^2$$

$$y = ax + b$$

3) $y = ax + b$

céges

$$\begin{aligned} a &= \frac{D_a}{D} \\ b &= \frac{D_b}{D} \end{aligned}$$

$$b = \frac{D_b}{D}$$

Leveretés: hennrealak
hatása a
betonműlőd-
ségra
v. építési
c. jegeket

$$D = \begin{vmatrix} \sum x^2 & \sum x \\ \sum x & n \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum xy & \sum x \\ \sum y & n \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum x^2 & \sum xy \\ \sum x & \sum y \end{vmatrix}$$

<http://www.betonopus.hu/notesz/kutyanelyelv/>
Részletesen lásd itt:
regressz-keplet-1.pdf

Origin at the end⁴ parabola

4) $y = ax^2 + bx$

$$y = ax^2 + bx$$

$$F = \sum (y - ax^2 - bx)^2$$

$$\frac{\partial F}{\partial a} = -2 \sum (y - ax^2 - bx)x^2 = 0$$

$$\frac{\partial F}{\partial b} = -2 \sum (y - ax^2 - bx)x = 0$$

$$\sum y - a \sum x^4 - b \sum x^3 = 0$$

$$\sum xy - a \sum x^3 - b \sum x^2 = 0$$

$$D = \begin{vmatrix} \sum x^4 & \sum x^3 \\ \sum x^3 & \sum x^2 \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum x^2 y & \sum x^3 \\ \sum xy & \sum x^2 \end{vmatrix}$$

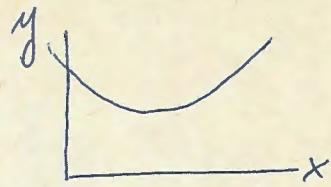
$$D_b = \begin{vmatrix} \sum x^4 & \sum x^2 y \\ \sum x^3 & \sum xy \end{vmatrix}$$

Meghatározandó:

$\sum x^2$	$\sum x^3$	$\sum x^4$
$\sum x^2 y$	$\sum xy$	
(1)	(1)	

5) $y = axx^2 + bx + c$

$$y = ax^2 + bx + c$$



$$a = \frac{D_a}{D}; \quad b = \frac{D_b}{D}; \quad c = \frac{D_c}{D}$$

Leveretek:
szemcselék
hatása a
betonműlárda-
ságra

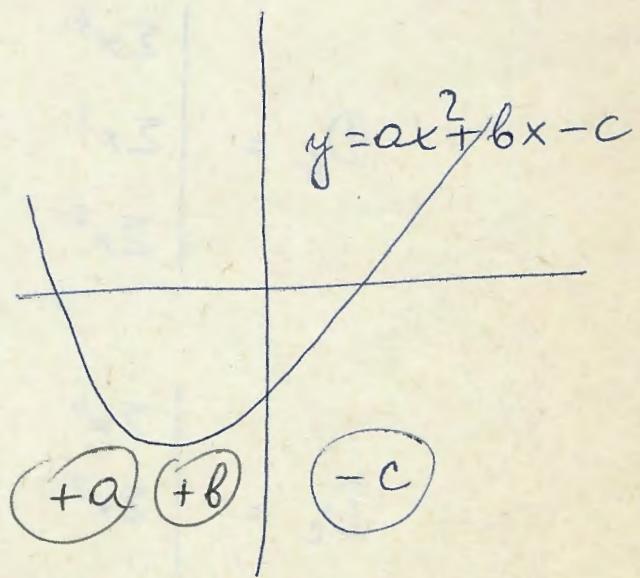
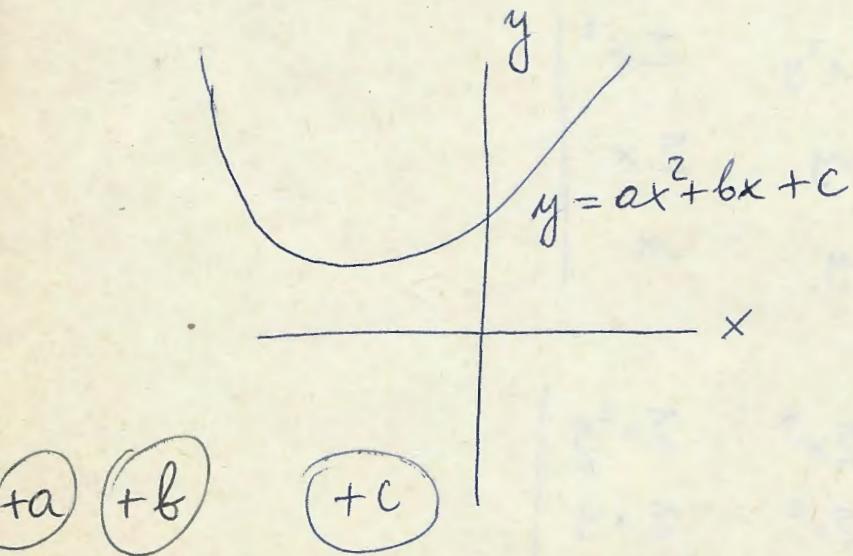
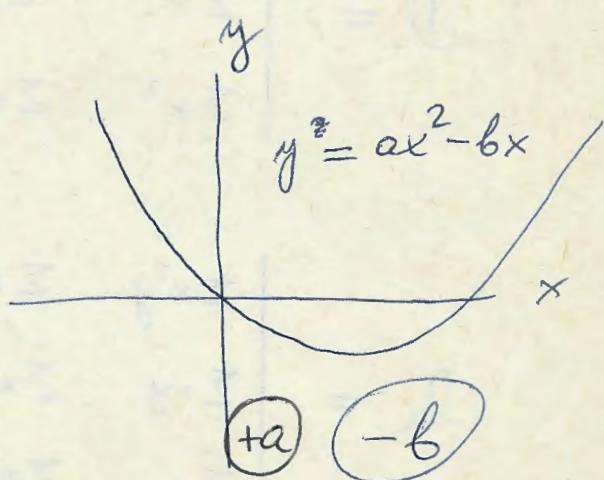
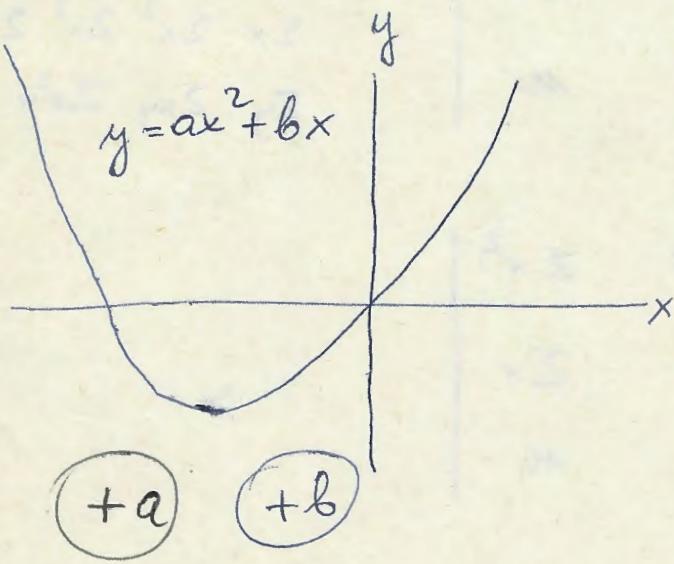
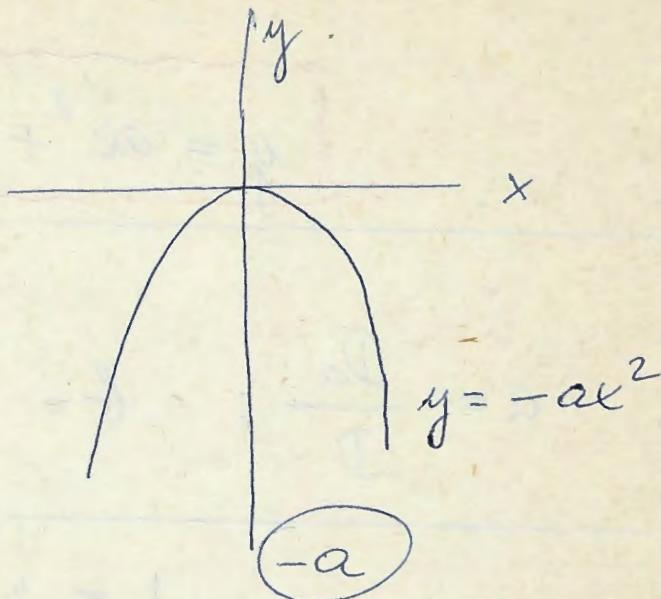
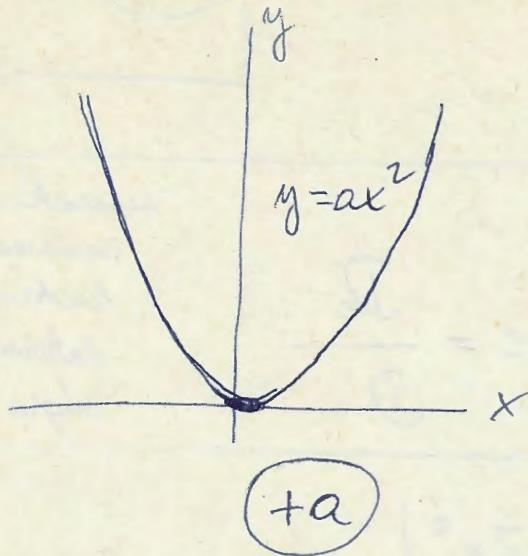
$$D = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & n \end{vmatrix} \quad \text{Meghatározandó:} \\ \sum x \quad \sum x^2 \quad \sum x^3 \quad \sum x^4 \\ \sum y \quad \sum xy \quad \sum x^2 y$$

$$D_a = \begin{vmatrix} \sum x^2 y & \sum x^3 & \sum x^2 \\ \sum xy & \sum x^2 & \sum x \\ \sum y & \sum x & n \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum x^4 & \sum x^2 y & \sum x^2 \\ \sum x^3 & \sum xy & \sum x \\ \sum x^2 & \sum y & n \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum x^2 y \\ \sum x^3 & \sum x^2 & \sum xy \\ \sum x^2 & \sum x & \sum y \end{vmatrix}$$

[Részletesen lásd itt:
http://www.betonopus.hu/notesz/kutyanelyelv/regressz-keplet-1.pdf](http://www.betonopus.hu/notesz/kutyanelyelv/regressz-keplet-1.pdf)



$$6) \quad y = ax^2 + bx + c$$

Három ponton átmenő parabola

$$\left. \begin{array}{l} y_1 = Ax_1^2 + Bx_1 + C \\ y_2 = Ax_2^2 + Bx_2 + C \\ y_3 = Ax_3^2 + Bx_3 + C \end{array} \right\}$$

$$\begin{matrix} m = x \\ a, b, c = y \end{matrix}$$

$$A = \frac{D_A}{D} \quad B = \frac{D_B}{D} \quad C = \frac{D_C}{D}$$

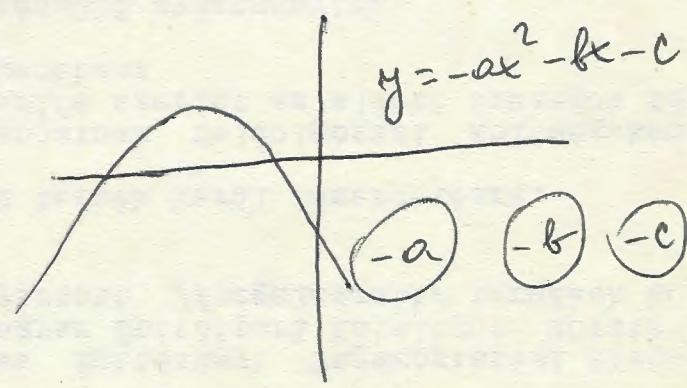
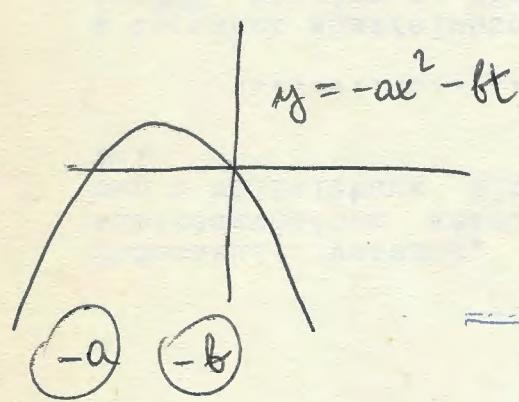
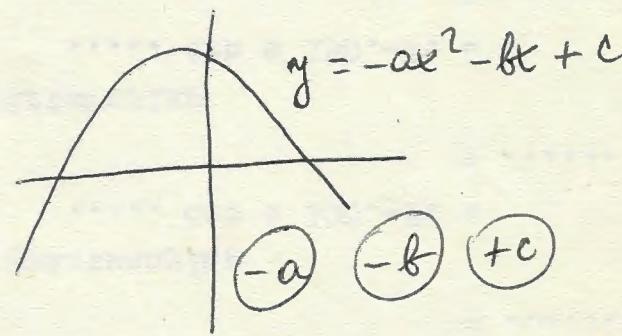
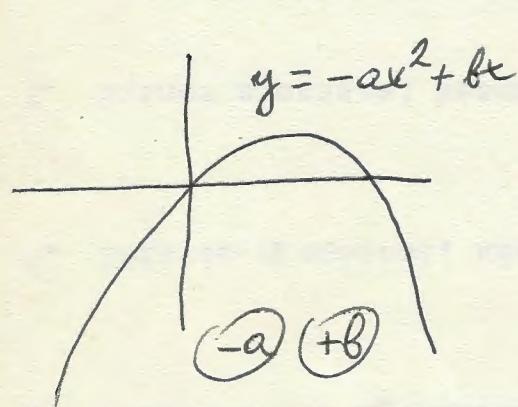
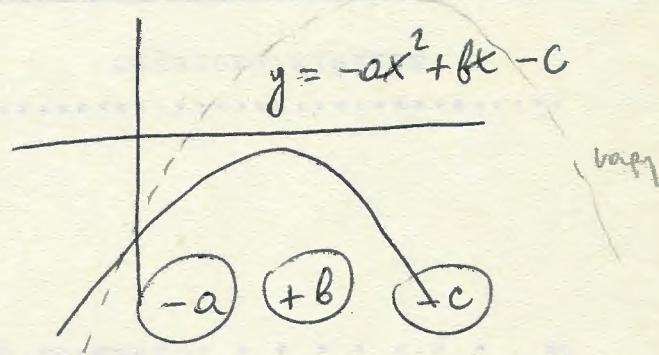
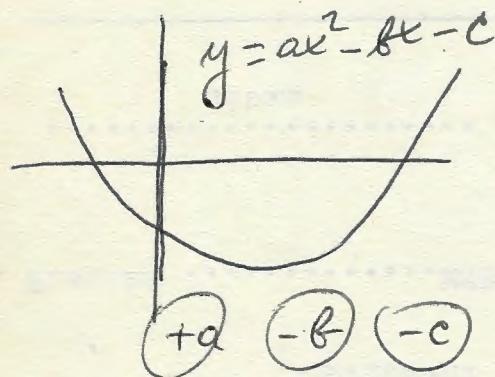
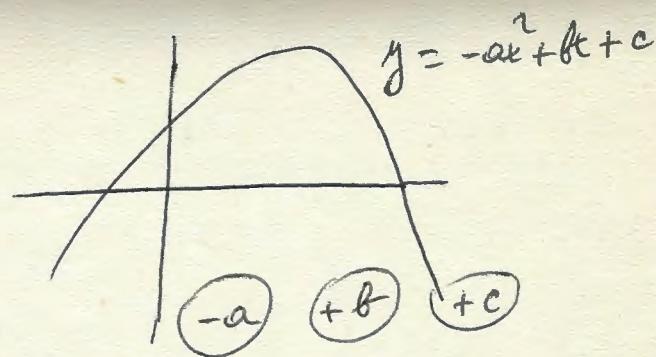
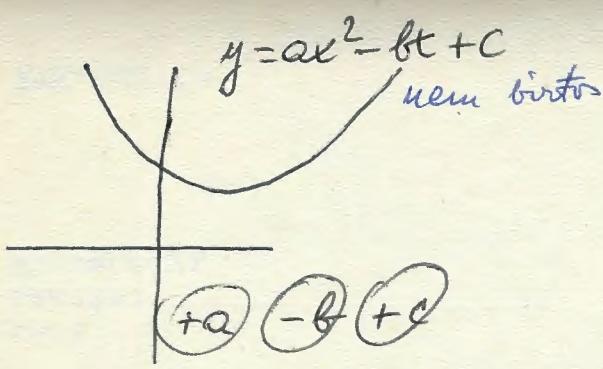
három egyenlet, három ismeretlen, determinánsval oldható meg:

$$D = \begin{vmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{vmatrix} = x_1^2 \underbrace{(x_2 - x_3)}_{x_1^2 \cdot D_{01}} - x_1 \underbrace{(x_2^2 - x_3^2)}_{-x_1 \cdot D_{02}} + \underbrace{(x_2^2 \cdot x_3 - x_2 \cdot x_3^2)}_{D_{03}}$$

$$D_A = \begin{vmatrix} y_1 & x_1 & 1 \\ y_2 & x_2 & 1 \\ y_3 & x_3 & 1 \end{vmatrix} = y_1(x_2 - x_3) - x_1(y_2 - y_3) + \underbrace{(y_2 \cdot x_3 - x_2 \cdot y_3)}_{y_1 \cdot D_{01} - x_1 \cdot D_{A2} + D_{A3}}$$

$$D_B = \begin{vmatrix} x_1^2 & y_1 & 1 \\ x_2^2 & y_2 & 1 \\ x_3^2 & y_3 & 1 \end{vmatrix} = x_1^2(y_2 - y_3) - y_1(x_2^2 - x_3^2) + \underbrace{(x_2^2 \cdot y_3 - y_2 \cdot x_3^2)}_{x_1^2 \cdot D_{A2} - y_1 \cdot D_{02} + D_{B3}}$$

$$D_C = \begin{vmatrix} x_1^2 & x_1 & y_1 \\ x_2^2 & x_2 & y_2 \\ x_3^2 & x_3 & y_3 \end{vmatrix} = x_1^2(x_2 y_3 - y_2 x_3) - x_1(x_2^2 y_3 - y_2 \cdot x_3^2) + y_1(x_2^2 \cdot x_3 - x_2 \cdot x_3^2) \\ x_1^2(-D_{A3}) - x_1 \cdot D_{B3} + y_1 D_{03}$$



$$b^2 - 4ac < 0$$

$$b^2 \leq 4ac$$

$$\therefore 0 < \frac{b^2}{4ac} < 1$$

$a < 0$ $c > 0$ arrows explain

also minima $x_1 < x_2$

$$y = ax^3 + bx^2 + cx$$

7) $y = ax^3 + bx^2 + cx$

origon átmenő harmadfokú parabola

G-E diagramról kiszámolás.

$$F = \sum_{i=1}^n (y_i - ax_i^3 - bx_i^2 - cx_i)^2 = \min$$

a továbbiakban az i indexet elhagyva

$$\frac{\partial F}{\partial a} = -2 \sum (y - ax^3 - bx^2 - cx)x^3 = 0$$

$$\frac{\partial F}{\partial b} = -2 \sum (y - ax^3 - bx^2 - cx)x^2 = 0$$

$$\frac{\partial F}{\partial c} = -2 \sum (y - ax^3 - bx^2 - cx)x = 0$$

$$\sum yx^3 - a \sum x^6 - b \sum x^5 - c \sum x^4 = 0$$

$$\sum yx^2 - a \sum x^5 - b \sum x^4 - c \sum x^3 = 0$$

$$\sum yx - a \sum x^4 - b \sum x^3 - c \sum x^2 = 0$$

$$D = \begin{vmatrix} \sum x^6 & \sum x^5 & \sum x^4 \\ \sum x^5 & \sum x^4 & \sum x^3 \\ \sum x^4 & \sum x^3 & \sum x^2 \end{vmatrix}$$

$$D_F = \begin{vmatrix} \sum x^6 & \sum yx^3 & \sum x^4 \\ \sum x^5 & \sum yx^2 & \sum x^3 \\ \sum x^4 & \sum yx & \sum x^2 \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum yx^3 & \sum x^5 & \sum x^4 \\ \sum yx^2 & \sum x^4 & \sum x^3 \\ \sum yx & \sum x^3 & \sum x^2 \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum x^6 & \sum x^5 & \sum yx^3 \\ \sum x^5 & \sum x^4 & \sum yx^2 \\ \sum x^4 & \sum x^3 & \sum yx \end{vmatrix}$$

$$\text{Meghatározandó} = \sum x^2 \quad \sum x^3 \quad \sum x^4 \quad \sum x^5 \quad \sum x^6 \quad \sum yx \quad \sum yx^2 \quad \sum yx^3$$

$$D = \sum x^6 \left(\underbrace{\sum x^2 \sum x^4 - \sum x^3 \sum x^3}_{D_{01}} \right) - \sum x^5 \left(\underbrace{\sum x^2 \sum x^5 - \sum x^3 \sum x^4}_{D_{02}} \right) +$$

$$+ \sum x^4 \left(\underbrace{\sum x^3 \sum x^5 - \sum x^4 \sum x^4}_{D_{03}} \right) = D_{01} \sum x^6 - D_{02} \sum x^5 + D_{03} \sum x^4$$

$$D_a = \sum yx^3 \cdot D_{01} - \sum x^5 \left(\underbrace{\sum yx^2 \cdot \sum x^2 - \sum yx \cdot \sum x^3}_{D_{12}} \right) +$$

$$+ \sum x^4 \left(\underbrace{\sum yx^2 \cdot \sum x^3 - \sum yx \cdot \sum x^4}_{D_{13}} \right) =$$

$$D_b = \sum x^6 \cdot D_{12} - \sum yx^3 \cdot D_{02} + \sum x^4 \left(\underbrace{\sum yx \cdot \sum x^5 - \sum yx^2 \cdot \sum x^4}_{D_{23}} \right)$$

$$D_c = \sum x^6 \cdot (-D_{13}) - \sum x^5 \cdot D_{23} + \sum yx^3 \cdot D_{03}$$

Meghatározandó:

$$\sum x^2 \quad \sum x^3 \quad \sum x^4 \quad \sum x^5 \quad \sum x^6 \quad \sum yx \quad \sum yx^2 \quad \sum yx^3$$

$$D_{01} = \sum x^2 \sum x^4 - \sum x^3 \sum x^3$$

$$D_{02} = \sum x^2 \sum x^5 - \sum x^3 \sum x^4$$

$$D_{03} = \sum x^3 \sum x^5 - \sum x^4 \sum x^4$$

$$D_{12} = \sum yx^2 \cdot \sum x^2 - \sum yx \cdot \sum x^3$$

$$D_{13} = \sum yx^2 \cdot \sum x^3 - \sum yx \cdot \sum x^4$$

$$D_{23} = \sum yx \cdot \sum x^5 - \sum yx^2 \sum x^4$$

$$D = D_{01} \sum x^6 - D_{02} \sum x^5 + D_{03} \sum x^4$$

$$D_a = D_{01} \sum yx^3 - D_{12} \sum x^5 + D_{13} \sum x^4$$

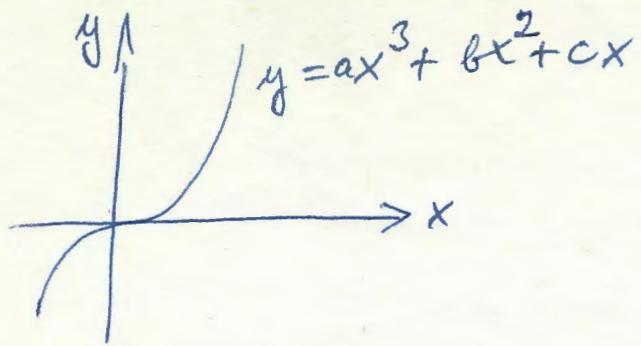
$$D_b = D_{12} \sum x^6 - D_{02} \sum yx^3 + D_{23} \sum x^4$$

$$D_c = -D_{13} \sum x^6 - D_{23} \sum x^5 + D_{03} \sum yx^3$$

$$a = \frac{D_a}{D}$$

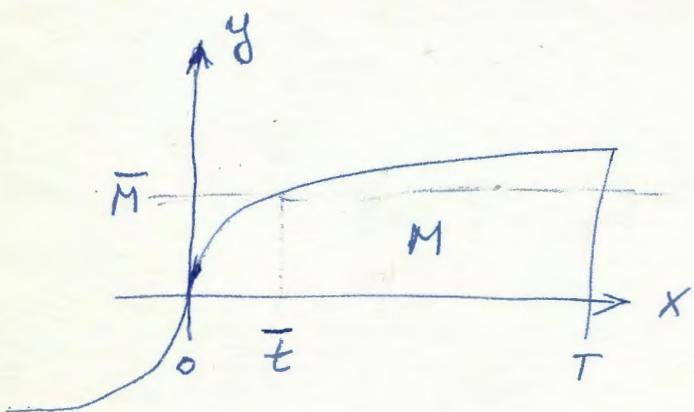
$$b = \frac{D_b}{D}$$

$$c = \frac{D_c}{D}$$



$x = \text{idő}''$
 $y = \text{alakváltozás}$

Amikor az inverze hellene:



$$x = ay^3 + by^2 + cy$$

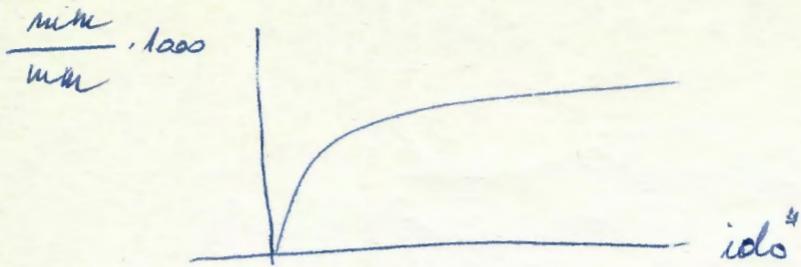
de ebből az y -t lefejerni
nincs lehetetlenség, ezért
az alábbi függvényel
hellene dolgozni, ami
megközelíti az inverz függést:

$$\begin{aligned} y &= a \cdot \sqrt[3]{x} + b \sqrt{x} + cx = \\ &= a \cdot x^{\frac{1}{3}} + b \cdot x^{\frac{1}{2}} + cx \end{aligned}$$

A görbe alatti terület $0 < x < T$ intervallumban:

$$\begin{aligned} M &= \int_0^T y dx = a \int_0^T x^{\frac{1}{3}} dx + b \int_0^T x^{\frac{1}{2}} dx + c \int_0^T x dx = \\ &= \frac{3}{4} a \cdot T^{\frac{4}{3}} + \frac{2}{3} b \cdot T^{\frac{3}{2}} + \frac{1}{2} c \cdot T^2 \end{aligned}$$

$$\begin{aligned} \bar{M} &= \frac{M}{T} \quad \text{probabilitás al meghatározandó } \bar{T}, \text{ amikor} \\ &\quad e = \bar{M} \cdot \bar{T} - \int_0^{\bar{T}} y dx \end{aligned}$$



Ha a görbe alatti területet megnövörök

$\frac{1000}{1000}$ - rel, ~~E_{max}~~

akkor m a henger magassága, akkor ezzel

mm.perc dimenziójú menetidőre jut, ami
 int x ido" (megkövülik a sejtszám)

$$y = a \cdot x^{\frac{1}{3}} + b \cdot x^{\frac{1}{2}} + c \cdot x$$

$$F = \sum (y - a \cdot x^{\frac{1}{3}} - b \cdot x^{\frac{1}{2}} - c \cdot x)^2 = \min$$

8) $y = axx^{(1/3)} + bx^{(1/2)} + cx$

$$\frac{\partial F}{\partial a} = -2 \sum (y - a \cdot x^{\frac{1}{3}} - b \cdot x^{\frac{1}{2}} - c \cdot x) \cdot x^{\frac{1}{3}} = 0$$

$$\frac{\partial F}{\partial b} = -2 \sum (y - a \cdot x^{\frac{1}{3}} - b \cdot x^{\frac{1}{2}} - c \cdot x) \cdot x^{\frac{1}{2}} = 0$$

$$\frac{\partial F}{\partial c} = -2 \sum (y - a \cdot x^{\frac{1}{3}} - b \cdot x^{\frac{1}{2}} - c \cdot x) \cdot x = 0$$

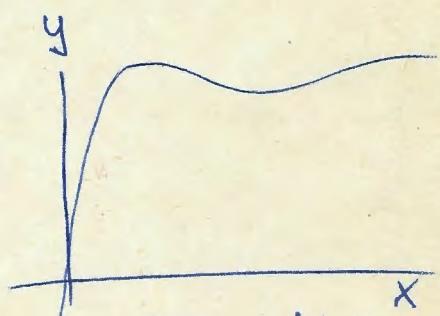
$$\sum x^{\frac{1}{3}} \cdot y - a \sum x^{\frac{2}{3}} - b \sum x^{\frac{5}{6}} - c \sum x^{\frac{4}{3}} = 0$$

$$\sum x^{\frac{1}{2}} \cdot y - a \sum x^{\frac{5}{6}} - b \sum x^{\frac{3}{2}} - c \sum x^{\frac{3}{2}} = 0$$

$$\sum xy - a \sum x^{\frac{4}{3}} - b \sum x^{\frac{3}{2}} - c \sum x^2 = 0$$

$$D = \begin{vmatrix} \sum x^{\frac{2}{3}} & \sum x^{\frac{5}{6}} & \sum x^{\frac{4}{3}} \\ \sum x^{\frac{5}{6}} & \sum x & \sum x^{\frac{3}{2}} \\ \sum x^{\frac{4}{3}} & \sum x^{\frac{3}{2}} & \sum x^2 \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum x^{\frac{1}{3}} \cdot y & \sum x^{\frac{5}{6}} & \sum x^{\frac{4}{3}} \\ \sum x^{\frac{1}{2}} \cdot y & \sum x & \sum x^{\frac{3}{2}} \\ \sum x \cdot y & \sum x^{\frac{3}{2}} & \sum x^2 \end{vmatrix}$$



föllt nöts-
ertere in
lehet

$$D_8 = \begin{vmatrix} \Sigma \times \frac{2}{3} & \Sigma \times \frac{1}{3} \cdot y & \Sigma \times \frac{4}{3} \\ \Sigma \times \frac{5}{6} & \Sigma \times \frac{1}{2} \cdot y & \Sigma \times \frac{3}{2} \\ \Sigma \times \frac{4}{3} & \Sigma \times y & \Sigma \times^2 \end{vmatrix}$$

$$D_6 = \begin{vmatrix} \Sigma \times \frac{2}{3} & \Sigma \times \frac{5}{6} & \Sigma \times \frac{1}{3} \cdot y \\ \Sigma \times \frac{5}{6} & \Sigma \times & \Sigma \times \frac{1}{2} \cdot y \\ \Sigma \times \frac{4}{3} & \Sigma \times \frac{3}{2} & \Sigma \times y \end{vmatrix}$$

$$\boxed{y = a \cdot x^b}$$

$$F = \sum (y - a \cdot x^b)^2$$

$$\frac{\partial F}{\partial a} = -2 \sum (y - a \cdot x^b) x^b = 0$$

$$\frac{\partial F}{\partial b} = -2 \sum (y - a \cdot x^b) a \frac{x^b}{\ln x} = 0$$

$$\sum y \cdot x^b - a \sum x^{2b} = 0 \quad \text{ez így nem meggy}$$

$$\sum y \frac{x^b}{\ln x} - a \sum \frac{x^{2b}}{\ln x} = 0$$

$$\lg y = \lg a + b \lg x \quad u = A + B \cdot v$$

$$\lg y = u \quad B = \frac{D_B}{D} \quad A = \frac{D_A}{D}$$

$$\lg x = v$$

$$\lg a = A$$

$$b = B$$

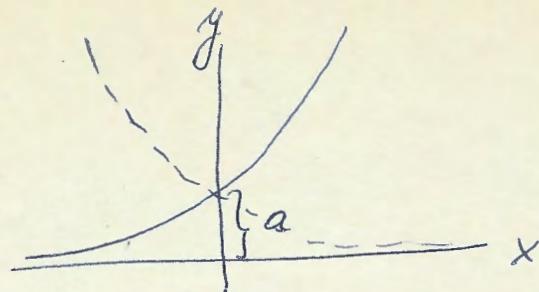
$$D = \begin{vmatrix} \sum v^2 & \sum v \\ \sum v & n \end{vmatrix}$$

$$D_B = \begin{vmatrix} \sum uv & \sum v \\ \sum u & n \end{vmatrix} \quad D_A = \begin{vmatrix} \sum v^2 & \sum uv \\ \sum v & \sum u \end{vmatrix}$$

Ez így már megoldható!

10) $y = a \cdot e^{bx}$

$$y = a \cdot e^{bx}$$



\times tengely aszimptótájú exponenciális függvény

$$e^b = c \quad y = a \cdot c^x \quad \lg y = \lg a + x \lg c$$

$$\lg y = u$$

$$\lg a = A$$

$$\lg c = C$$

$$u = CX + A$$

$$C = \frac{D_C}{D}; \quad A = \frac{D_A}{D}$$

$$c = 10^C$$

$$a = 10^A$$

$$b = \ln c = 2,3026, G$$

Leveretés: 1973 elői Berná&félék

utpalya eredményi téma

nakirítése

\hookrightarrow Előkészületek a praktikum 2. jeppel

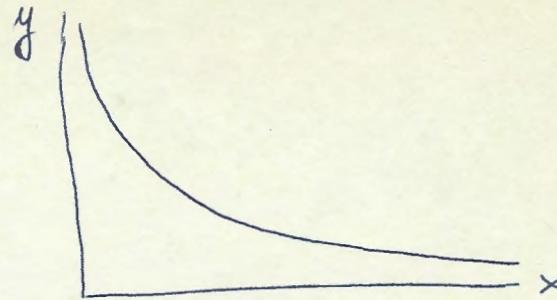
$$D = \begin{vmatrix} \sum x^2 & \sum x \\ \sum x & n \end{vmatrix}$$

$$D_C = \begin{vmatrix} \sum x \lg y & \sum x \\ \sum \lg y & n \end{vmatrix}$$

$$D_A = \begin{vmatrix} \sum x^2 & \sum x \lg y \\ \sum x & \sum \lg y \end{vmatrix}$$

11) $y = 1/(ax)$

$$y = \frac{1}{ax}$$



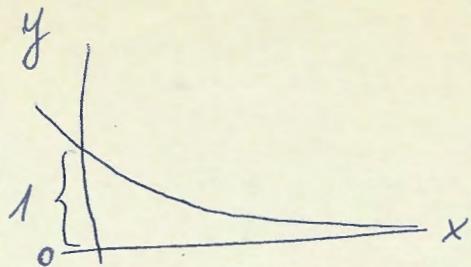
x és y tengelyen általáztatott hiperbola

$$a = \frac{\sum \frac{x}{y}}{\sum x^2}$$

Leveretek: 1973 előtt Bernárd-féle utpalja eredményi téma mérésekre

12) $y = 1/(ax+1)$

$$y = \frac{1}{ax + 1}$$



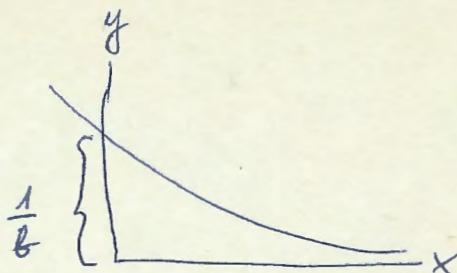
x tengely asszimptótajú hiperbola

$$a = \frac{\sum \frac{x}{y} - \bar{x}}{\sum x^2}$$

Leveretés: 1973. évi Bernák-féle útfálya érdeseképi téma munkálképe

13) $y = 1/(ax+b)$

$$\boxed{y = \frac{1}{ax+b}}$$



x tengely asymptotikus hiperbola

$$a = \frac{D_a}{D} \quad i \quad b = \frac{D_b}{D}$$

$$D = \begin{vmatrix} \sum x^2 & \sum x \\ \sum x & n \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum \frac{x}{y} & \sum x \\ \sum \frac{1}{y} & n \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum x^2 & \sum \frac{x}{y} \\ \sum x & \sum \frac{1}{y} \end{vmatrix}$$

Meghatározandó:

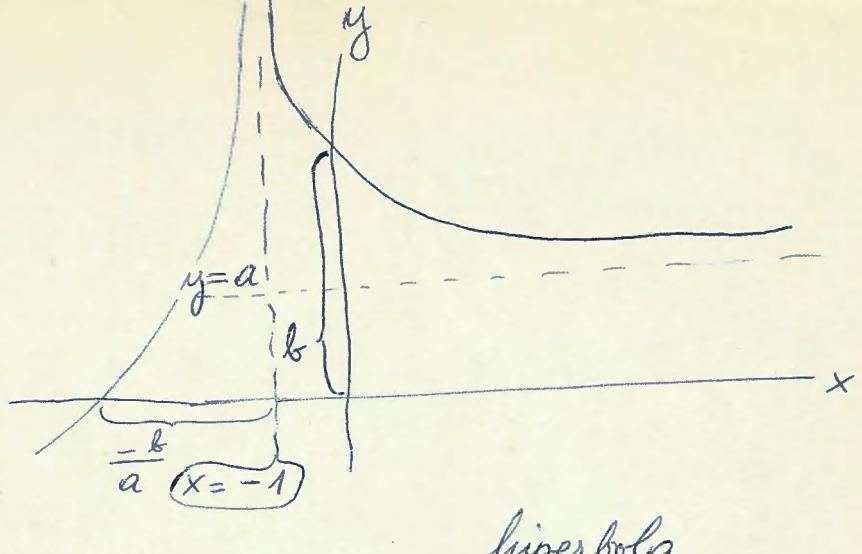
$$\sum x \quad \sum x^2 \quad \sum \frac{x}{y} \quad \sum \frac{1}{y}$$

Ez a független érték, amikor a mérés eredmények között $y_i = 0$ is előfordul, nem használható, mert zeroval nem tudunk osztani, tehát az $\frac{1}{y}$ és $\frac{x}{y}$ nem adhatók!

Levereté: 1973. évi Rennák-féle útpálya érdekes téma munkájára

14) $y = (ax+b)/(x+1)$

$$y = \frac{ax+b}{x+1}$$



hiperbola

$$a = \frac{D_a}{D} \quad ; \quad b = \frac{D_b}{D}$$

$$D = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \frac{x}{(x+1)^2} \\ \sum \frac{x}{(x+1)^2} & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

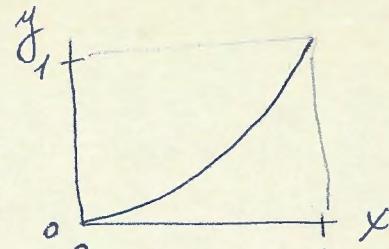
$$D_a = \begin{vmatrix} \sum \frac{xy}{x+1} & \sum \frac{x}{(x+1)^2} \\ \sum \frac{y}{x+1} & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \frac{xy}{x+1} \\ \sum \frac{x}{(x+1)^2} & \sum \frac{y}{x+1} \end{vmatrix}$$

Levereth : 1973. évi Pernáth-féle utpalja érdekeségi téma makettje

15) $y = (axx)/(x+a-1)$

$$y = \frac{ax}{x+a-1}$$



A $P(0,0)$ és $P(1,1)$ pontokon átmenő hiperbola

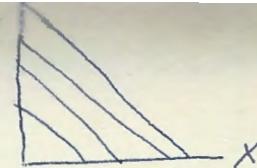
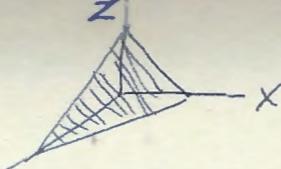
$$a = \frac{n - 2 \sum x + \sum x^2}{n + \sum \frac{x^2}{y} - \sum x - \sum \frac{x}{y}}$$

levezetés: összefüggés a szerothó- és haviestermelhet nemalakja
 és Los Angeles virágálat merinti aprózódási rendszerége körött.

$$16) z = axy + bxy + c$$

$$z = ay + bx + c$$

ferde



sik, ill. párhuzamos egyenesek
(a síkban)

$$a = \frac{D_a}{D}, \quad b = \frac{D_b}{D}, \quad c = \frac{D_c}{D}$$

levezetés: Bolyaihoz
működés és hármasok
nemelőzetességek
analitikus függvénye

$$D = \begin{vmatrix} \sum y^2 & \sum xy & \sum y \\ \sum xy & \sum x^2 & \sum x \\ \sum y & \sum x & n \end{vmatrix}$$

1997. július 3, COLAS
(Cseh Zoltán) keretében
a vizes Devecsei hő-
szabályozás értelemezése
lással BO - 20 floppy.

$$D_a = \begin{vmatrix} \sum zy & \sum xy & \sum y \\ \sum zx & \sum x^2 & \sum x \\ \sum z & \sum x & n \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum y^2 & \sum zy & \sum y \\ \sum xy & \sum zx & \sum x \\ \sum y & \sum z & n \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum y^2 & \sum xy & \sum zy \\ \sum xy & \sum x^2 & \sum zx \\ \sum y & \sum x & \sum z \end{vmatrix}$$

$$z = ax^2 + bx + y + c$$

$$= \Delta = (y_2 - y_1)$$

$$\Delta = (y_2 - y_1)$$

17) $z = ax^2 + bx + y + c$

ha az "x"-re a-val pl. nő, akkor a "z" értéke is Δ-val nő

egymással párhuzamos másodfokú parabolák

$$a = \frac{D_a}{D}; \quad b = \frac{D_b}{D}; \quad c = \frac{D_c}{D}$$

$$D = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & n \end{vmatrix}$$

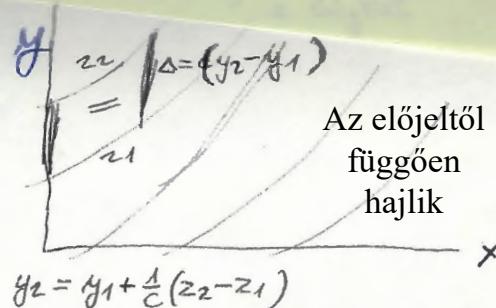
$$D_a = \begin{vmatrix} \sum (z-y)x^2 & \sum x^3 & \sum x^2 \\ \sum (z-y)x & \sum x^2 & \sum x \\ \sum (z-y) & \sum x & n \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum x^4 & \sum (z-y)x^2 & \sum x^2 \\ \sum x^3 & \sum (z-y)x & \sum x \\ \sum x^2 & \sum (z-y) & n \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum (z-y)x^2 \\ \sum x^3 & \sum x^2 & \sum (z-y)x \\ \sum x^2 & \sum x & \sum (z-y) \end{vmatrix}$$

$$z = ax^2 + bx + cy + d$$

Egy másik parabolára vonatkozóan hasonlóképpen parabolával
a réthban. Ha a z_1 értéke z_2 -re nő, (a növekmény $z_2 - z_1$)
akkor az y_2 értéke $\frac{1}{c}(z_2 - z_1)$ értékkel növeknik meg, az új érték $y_2 = y_1 + \frac{1}{c}(z_2 - z_1)$



$$a = \frac{D_a}{D}; \quad b = \frac{D_b}{D}; \quad c = -\frac{D_c}{D}; \quad d = \frac{D_d}{D}$$

$$D_5 \quad D = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum x^2 y & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum xy & \sum x \\ \sum x^2 y & \sum xy & \sum y^2 & \sum y \\ \sum x^2 & \sum x & \sum y & n \end{vmatrix} \quad \text{Meghatározandó: } \begin{matrix} \sum x^4 \\ \sum x^3 \\ \sum x^2 \\ \sum x \\ \sum y \end{matrix}$$

$$D_1 \quad D_a = \begin{vmatrix} \sum x^2 z & \sum x^3 & \sum x^2 y & \sum x^2 \\ \sum xz & \sum x^2 & \sum xy & \sum x \\ \sum yz & \sum xy & \sum y^2 & \sum y \\ \sum z & \sum x & \sum y & n \end{vmatrix} \quad \begin{matrix} \sum y \\ \sum x^2 y \\ \sum x^2 z \\ \sum xz \end{matrix}$$

$$D_2 \quad D_b = \begin{vmatrix} \sum x^4 & \sum x^2 z & \sum x^2 y & \sum x^2 \\ \sum x^3 & \sum xz & \sum xy & \sum x \\ \sum x^2 y & \sum yz & \sum y^2 & \sum y \\ \sum x^2 & \sum z & \sum y & n \end{vmatrix} \quad \begin{matrix} \sum y^2 \\ \sum z \end{matrix}$$

$$D_3 \quad D_c = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum x^2 z & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum xz & \sum x \\ \sum x^2 y & \sum xy & \sum yz & \sum y \\ \sum x^2 & \sum x & \sum z & n \end{vmatrix}$$

$$D_4 \quad D_d = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum x^2 y & \sum x^2 z \\ \sum x^3 & \sum x^2 & \sum xy & \sum xz \\ \sum x^2 y & \sum xy & \sum y^2 & \sum yz \\ \sum x^2 & \sum x & \sum y & \sum z \end{vmatrix}$$

$$18) \quad z = a \times x^2 + b \times x + c \times y + d$$

$$D_a = (D_1) = \sum x^2 z \cdot D_4 - \sum x^3 \cdot D_1 + \sum x^2 y \cdot D_2 - \sum x^2 \cdot D_3$$

$$L = ax^2 + bx + cy + d$$

$$D_b = (D_2) = \sum x^4 \cdot D_1 - \sum x^2 z \cdot D_5 + \sum x^2 y \cdot D_8 - \sum x^2 \cdot D_9$$

$$D_c = (D_3) = -\sum x^4 \cdot D_2 - \sum x^3 \cdot D_8 + \sum x^2 z \cdot D_6 - \sum x^2 \cdot D_{10}$$

$$D_d = (D_4) = \sum x^4 \cdot D_3 + \sum x^3 \cdot D_9 + \sum x^2 y \cdot D_{10} - \sum x^2 z \cdot D_7$$

$$D = (D_0) = \sum x^4 \cdot D_4 - \sum x^3 \cdot D_5 + \sum x^2 y \cdot D_6 - \sum x^2 \cdot D_7$$

$$D_{\phi 5} = \begin{vmatrix} \sum x^4 & \sum x^3 & \sum xy & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum xy & \sum x \\ \sum x^2 y & \sum xy & \sum y^2 & \sum y \\ \sum x^2 & \bar{x} & \bar{y} & 10+n \end{vmatrix}$$

$\textcircled{D}_{\phi 5} = \sum x^4 D_{\phi 1} - \sum x^3 D_{\phi 2} + \sum xy D_{\phi 3} - \sum x^2 D_{\phi 4}$

\rightarrow $D_{\phi 1} = \begin{vmatrix} \sum x^2 & \sum xy & \sum x & \sum x^2 & \sum xy \\ \sum xy & \sum y^2 & \sum y & \sum xy & \sum y^2 \\ \sum x & \sum y & 10+n & \sum x & \sum y \end{vmatrix}$

$$D_{\phi 1} = \sum x^2 (10 \sum y^2 - \sum y \sum y) - \sum xy (10 \sum xy - \sum x \sum y) + \sum x (\sum xy \sum y - \sum y^2 \sum x)$$

\rightarrow $D_{\phi 2} = \begin{vmatrix} \sum x^3 & \sum xy & \sum x & \sum x^3 & \sum xy \\ \sum x^2 y & \sum y^2 & \sum y & \sum x^2 y & \sum y^2 \\ \sum x^2 & \sum y & 10+n & \sum x^2 & \sum y \end{vmatrix}$

$$D_{\phi 2} = \sum x^3 (10 \sum y^2 - \sum y \sum y) - \sum xy (10 \sum x^2 y - \sum y \sum x^2) + \sum x (\sum x^2 y \sum y - \sum y^2 \sum x^2)$$

\rightarrow $D_{\phi 3} = \begin{vmatrix} \sum x^3 & \sum x^2 & \sum x & \sum x^3 & \sum x^2 \\ \sum x^2 y & \sum xy & \sum y & \sum x^2 y & \sum xy \\ \sum x^2 & \sum x & 10+n & \sum x^2 & \sum x \end{vmatrix}$

$$D_{\phi 3} = \sum x^3 (10 \sum xy - \sum x \sum y) - \sum x^2 (10 \sum x^2 y - \sum x^2 \sum y) + \sum x (\sum x^2 y \sum x - \sum x^2 \sum xy)$$

\rightarrow $D_{\phi 4} = \begin{vmatrix} \sum x^3 & \sum x^2 & \sum xy & \sum x^3 & \sum x^2 \\ \sum x^2 y & \sum xy & \sum y^2 & \sum x^2 y & \sum xy \\ \sum x^2 & \sum x & \sum y & \sum x^2 & \sum x \end{vmatrix}$

$$D_{\phi 4} = \sum x^3 (\sum xy \cdot \sum y - \sum y^2 \sum x) - \sum x^2 (\sum x^2 y \cdot \sum y - \sum y^2 \sum x^2) + \sum xy (\sum x^2 y \cdot \sum x - \sum xy \cdot \sum x^2)$$

$$D_1 = \begin{vmatrix} \sum x^2 z & \sum x^3 & \sum x^2 y & \sum x^2 \\ \bar{z} x z & \sum x^2 & \bar{z} x y & \bar{z} x \\ \bar{z} y z & \sum x y & \bar{z} y^2 & \bar{z} y \\ \bar{z} z & \sum x & \bar{z} y & 10 \end{vmatrix}$$

$$D_1 = \sum x^2 D_{11} - \sum x^3 D_{12} + \sum x^2 y D_{13} - \sum x^2 D_{14}$$

$$D_{11} = \begin{vmatrix} \sum x^2 & \sum x y & \bar{z} x \\ \sum x y & \sum y^2 & \bar{z} y \\ \bar{z} x & \bar{z} y & 10 \end{vmatrix} = D_{\phi 1}$$

$$D_{11} = \sum x^2 (10 \sum y^2 - \bar{z} y \bar{z} y) - \sum x y (10 \sum x y - \bar{z} x \bar{z} y) + \bar{z} x (\sum x y \bar{z} y - \bar{z} y^2 \sum x)$$

$$\xrightarrow{\text{XYZ(1)}} D_{12} = \begin{vmatrix} \bar{z} x z & \bar{z} x y & \bar{z} x \\ \bar{z} y z & \bar{z} y^2 & \bar{z} y \\ \bar{z} z & \bar{z} y & 10 \end{vmatrix} \begin{matrix} \bar{z} x z & \bar{z} x y \\ \bar{z} y z & \bar{z} y^2 \\ \bar{z} z & \bar{z} y \end{matrix}$$

X Y Z (1)

$$D_{12} = \sum x z (10 \bar{z} y^2 - \bar{z} y \bar{z} y) - \sum x y (10 \sum y z - \bar{z} y \bar{z} z) + \bar{z} x (\sum y z \bar{z} y - \bar{z} y^2 \bar{z} z)$$

$$\xrightarrow{2} D_{13} = \begin{vmatrix} \bar{z} x z & \bar{z} x^2 & \bar{z} x \\ \bar{z} y z & \bar{z} x y & \bar{z} y \\ \bar{z} z & \bar{z} x & 10 \end{vmatrix} \begin{matrix} \bar{z} x z & \bar{z} x^2 \\ \bar{z} y z & \bar{z} x y \\ \bar{z} z & \bar{z} x \end{matrix}$$

$$D_{13} = \sum x z (10 \sum x y - \bar{z} y \bar{z} x) - \sum x^2 (10 \sum y z - \bar{z} y \bar{z} z) + \bar{z} x (\sum y z \bar{z} x - \bar{z} x \bar{z} z)$$

$$\xrightarrow{3} D_{14} = \begin{vmatrix} \bar{z} x z & \bar{z} x^2 & \bar{z} x y \\ \bar{z} y z & \bar{z} x y & \bar{z} y^2 \\ \bar{z} z & \bar{z} x & \bar{z} y \end{vmatrix} \begin{matrix} \bar{z} x z & \bar{z} x^2 \\ \bar{z} y z & \bar{z} x y \\ \bar{z} z & \bar{z} x \end{matrix}$$

$$D_{14} = \sum x z (\sum x y \bar{z} y - \bar{z} x \bar{z} y^2) - \sum x^2 (\sum y z \bar{z} y - \bar{z} z \bar{z} y^2) + \bar{z} x (\sum y z \bar{z} x - \bar{z} z \bar{z} x y)$$

-3 -

$$D_2 = \begin{vmatrix} \sum x^4 & \sum x^2 z & \sum x^2 y & \sum x^2 \\ \sum x^3 & \sum xz & \sum xy & \sum x \\ \sum x^2 y & \sum yz & \sum y^2 & \sum y \\ \sum x^2 z & \sum z & \sum y & 10 \end{vmatrix}$$

$$D_2 = \sum x^4 D_{21} - \sum x^2 z D_{22} + \sum x^2 y D_{23} - \sum x^2 z D_{24}$$

$$D_{21} = \begin{vmatrix} \sum xz & \sum xy & \sum x \\ \sum yz & \sum y^2 & \sum y \\ \sum z & \sum y & 10 \end{vmatrix} = D_{12}$$

$$D_{21} = \sum xz (\sum y^2 - \sum y \cdot \sum y) - \sum xy (\sum yz - \sum y \cdot \sum z) + \sum x (\sum yz \cdot \sum y - \sum y^2 \cdot \sum z)$$

$$D_{22} = \begin{vmatrix} \sum x^3 & \sum xy & \sum x \\ \sum x^2 y & \sum y^2 & \sum y \\ \sum x^2 z & \sum y & 10 \end{vmatrix} = D_{02}$$

$$D_{22} = \sum x^3 (\sum y^2 - \sum y \cdot \sum y) - \sum xy (\sum x^2 y - \sum y \cdot \sum x^2) + \sum x (\sum x^2 y \cdot \sum y - \sum y^2 \cdot \sum x^2)$$

$$\rightarrow 8 \quad D_{23} = \begin{vmatrix} \sum x^3 & \sum xz & \sum x \\ \sum x^2 y & \sum yz & \sum y \\ \sum x^2 z & \sum z & 10 \end{vmatrix} = \begin{vmatrix} \sum x^3 & \sum xz \\ \sum x^2 y & \sum yz \\ \sum x^2 z & \sum z \end{vmatrix}$$

$$D_{23} = \sum x^3 (\sum yz - \sum z \cdot \sum y) - \sum xz (\sum x^2 y - \sum x \cdot \sum x^2) + \sum x (\sum x^2 y \cdot \sum z - \sum x^2 \cdot \sum yz)$$

$$\rightarrow 9 \quad D_{24} = \begin{vmatrix} \sum x^3 & \sum xz & \sum xy \\ \sum x^2 y & \sum yz & \sum y^2 \\ \sum x^2 z & \sum z & \sum y \end{vmatrix} = \begin{vmatrix} \sum x^3 & \sum xz \\ \sum x^2 y & \sum yz \\ \sum x^2 z & \sum z \end{vmatrix}$$

$$D_{24} = \sum x^3 (\sum yz \cdot \sum y - \sum z \cdot \sum y^2) - \sum xz (\sum x^2 y \cdot \sum y - \sum x \cdot \sum x^2 y^2) + \sum xy (\sum x^2 y \cdot \sum z - \sum x^2 \cdot \sum yz)$$

$$D_3 = \begin{vmatrix} \Sigma x^4 & \Sigma x^3 & \Sigma x^2 z & \Sigma x^2 \\ \Sigma x^3 & \Sigma x^2 & \Sigma x z & \Sigma x \\ \Sigma x^2 y & \Sigma x y & \Sigma y z & \Sigma y \\ \Sigma x^2 & \Sigma x & \Sigma z & 10 \end{vmatrix} = -4$$

$$D_3 = \Sigma x^4 D_{31} - \Sigma x^3 D_{32} + \Sigma x^2 z D_{33} - \Sigma x^2 D_{34}$$

$$D_{31} = \begin{vmatrix} \Sigma x^2 & \Sigma x z & \Sigma x \\ \Sigma x y & \Sigma y z & \Sigma y \\ \Sigma x & \Sigma z & 10 \end{vmatrix} = -D_{13}$$

$$D_{31} = \Sigma x^2 (10 \Sigma y z - \Sigma z \Sigma y) - \Sigma x z (\cancel{10} \Sigma x y - \Sigma x \Sigma y) + \Sigma x (\Sigma x y \Sigma z - \Sigma x \Sigma y z)$$

$$D_{32} = \begin{vmatrix} \Sigma x^3 & \Sigma x z & \Sigma x \\ \Sigma x^2 y & \Sigma y z & \Sigma y \\ \Sigma x^2 & \Sigma z & 10 \end{vmatrix} = D_{23}$$

$$D_{32} = \Sigma x^3 (\cancel{10} \Sigma y z - \Sigma z \Sigma y) - \Sigma x z (\cancel{10} \Sigma x^2 y - \Sigma x^2 \Sigma y) + \Sigma x (\Sigma x^2 y \Sigma z - \Sigma x^2 \Sigma y z)$$

$$D_{33} = \begin{vmatrix} \Sigma x^3 & \Sigma x^2 & \Sigma x \\ \Sigma x^2 y & \Sigma x y & \Sigma y \\ \Sigma x^2 & \Sigma x & 10 \end{vmatrix} = D_{\phi 3}$$

$$D_{33} = \Sigma x^3 (\cancel{10} \Sigma x y - \Sigma x \Sigma y) - \Sigma x^2 (\cancel{10} \Sigma x^2 y - \Sigma x^2 \Sigma y) + \Sigma x (\Sigma x^2 y \Sigma x - \Sigma x^2 \Sigma x y)$$

$$D_{34} = \begin{vmatrix} \Sigma x^3 & \Sigma x^2 & \Sigma x z \\ \Sigma x^2 y & \Sigma x y & \Sigma y z \\ \Sigma x^2 & \Sigma x & \Sigma z \end{vmatrix} \rightarrow 10$$

$$D_{34} = \Sigma x^3 (\Sigma x y \Sigma z - \Sigma x \Sigma y z) - \Sigma x^2 (\Sigma x^2 y \Sigma z - \Sigma x^2 \Sigma y z) + \Sigma x z (\Sigma x^2 y \Sigma x - \Sigma x^2 \Sigma x y)$$

$$D_4 = \begin{vmatrix} \Sigma x^4 & \Sigma x^3 & \Sigma x^2 y & \Sigma x^2 z \\ \Sigma x^3 & \Sigma x^2 & \Sigma xy & \Sigma xz \\ \Sigma x^2 y & \Sigma xy & \Sigma y^2 & \Sigma yz \\ \Sigma x^2 z & \Sigma xz & \Sigma yz & \Sigma z^2 \end{vmatrix}$$

$$D_4 = \Sigma x^4 D_{41} - \Sigma x^3 D_{42} + \Sigma x^2 y D_{43} - \Sigma x^2 z D_{44}$$

$$D_{41} = \begin{vmatrix} \Sigma x^2 & \Sigma xy & \Sigma xz \\ \Sigma xy & \Sigma y^2 & \Sigma yz \\ \Sigma x & \Sigma y & \Sigma z \end{vmatrix} = D_{14}$$

$$D_{41} = \Sigma x^2 (\Sigma y^2 \Sigma z - \Sigma y \Sigma yz) - \Sigma xy (\Sigma xy \Sigma z - \Sigma x \Sigma yz) + \Sigma xz (\Sigma xy \Sigma y - \Sigma x \Sigma y^2)$$

$$D_{42} = \begin{vmatrix} \Sigma x^3 & \Sigma xy & \Sigma xz \\ \Sigma x^2 y & \Sigma y^2 & \Sigma yz \\ \Sigma x^2 & \Sigma y & \Sigma z \end{vmatrix} = -D_{24}$$

$$D_{42} = \Sigma x^3 (\Sigma y^2 \Sigma z - \Sigma y \Sigma yz) - \Sigma xy (\Sigma x^2 y \Sigma z - \Sigma x^2 \Sigma yz) + \Sigma xz (\Sigma x^2 y \Sigma y - \Sigma x^2 \Sigma y^2)$$

$$D_{43} = \begin{vmatrix} \Sigma x^3 & \Sigma x^2 & \Sigma xz \\ \Sigma x^2 y & \Sigma xy & \Sigma yz \\ \Sigma x^2 & \Sigma x & \Sigma z \end{vmatrix} = D_{34}$$

$$D_{43} = \Sigma x^3 (\Sigma xy \Sigma z - \Sigma x \Sigma yz) - \Sigma x^2 (\Sigma x^2 y \Sigma z - \Sigma x^2 \Sigma yz) + \Sigma xz (\Sigma x^2 y \Sigma x - \Sigma x^2 \Sigma xy)$$

$$D_{44} = \begin{vmatrix} \Sigma x^3 & \Sigma x^2 & \Sigma xy \\ \Sigma x^2 y & \Sigma xy & \Sigma y^2 \\ \Sigma x^2 & \Sigma x & \Sigma y \end{vmatrix} = D_{\phi 4}$$

$$D_{44} = \Sigma x^3 (\Sigma xy \Sigma y - \Sigma y^2 \Sigma x) - \Sigma x^2 (\Sigma x^2 y \Sigma y - \Sigma y^2 \Sigma x^2) + \Sigma xy (\Sigma x^2 y \Sigma x - \Sigma xy \Sigma x^2)$$

$$a_1 = \sum x$$

$$a_2 = \sum x^2$$

$$a_3 = \sum x^3$$

$$a_4 = \sum x^4$$

$$a_5 = \sum y$$

$$a_6 = \sum y^2$$

$$a_7 = \sum xy$$

$$a_8 = \sum x^2y$$

$$a_9 = \underline{\underline{m}}$$

$$\sum z$$

$$\sum xz$$

$$\sum x^2z$$

$$\sum yz$$

3de van hove

a voorbereid in.

(33 - 35 program)

$$z = ax^2 + bx + cy + d$$

(73 - 32 program)

$$D_0 = \sum x^4 D_{01} - \sum x^3 D_{02} + \sum x^2 y D_{03} - \sum x^2 D_{04} =$$

$$= a_4 \cdot D_{01} - a_3 \cdot D_{02} + a_2 \cdot D_{03} - a_2 \cdot D_{04}$$

— · —

$$D_{01} = a_2 \cdot a_6 \cdot a_9 + a_7 \cdot a_5 \cdot a_1 + a_1 \cdot a_7 \cdot a_5 -$$

felirni

$$- a_1 \cdot a_6 \cdot a_1 - a_5 \cdot a_5 \cdot a_2 - a_9 \cdot a_7 \cdot a_7$$

$$D_{02} = a_3 \cdot a_6 \cdot a_8 + a_7 \cdot a_5 \cdot a_2 + a_1 \cdot a_8 \cdot a_5 -$$

felirni

$$- a_2 \cdot a_6 \cdot a_1 - a_5 \cdot a_5 \cdot a_3 - a_9 \cdot a_8 \cdot a_7$$

$$D_{03} = a_3 \cdot a_7 \cdot a_8 + a_2 \cdot a_5 \cdot a_2 + a_1 \cdot a_8 \cdot a_1 -$$

felirni

$$- a_2 \cdot a_7 \cdot a_1 - a_1 \cdot a_5 \cdot a_3 - a_9 \cdot a_8 \cdot a_2$$

$$D_{04} = a_3 \cdot a_7 \cdot a_5 + a_2 \cdot a_6 \cdot a_2 + a_7 \cdot a_8 \cdot a_1 -$$

felirni

$$- a_2 \cdot a_7 \cdot a_7 - a_1 \cdot a_6 \cdot a_3 - a_5 \cdot a_8 \cdot a_2$$

— · —

$$D_{01} = a_6(a_2 \cdot a_9 - a_1 \cdot a_1) + a_5(a_7 \cdot a_1 - a_5 \cdot a_2) + a_7(a_1 \cdot a_5 - a_9 \cdot a_7)$$

$$D_{02} = a_6(a_3 \cdot a_9 - a_2 \cdot a_1) + a_5(a_7 \cdot a_2 - a_5 \cdot a_3) + a_8(a_1 \cdot a_5 - a_9 \cdot a_7)$$

$$D_{03} = a_7(a_3 \cdot a_9 - a_2 \cdot a_1) + a_5(a_2 \cdot a_2 - a_1 \cdot a_3) + a_8(a_1 \cdot a_1 - a_9 \cdot a_2)$$

$$D_{04} = a_7(a_3 \cdot a_5 - a_1 \cdot a_7) + a_6(a_2 \cdot a_2 - a_1 \cdot a_3) + a_8(a_7 \cdot a_1 - a_5 \cdot a_2)$$

- 3 -

$$\begin{aligned} D_1 &= \Sigma x^2 z \cdot D_{11} - a_3 \cdot D_{12} + a_8 \cdot D_{13} - a_2 \cdot D_{14} = \\ &= \Sigma x^2 z \cdot D_{01} - a_3 \cdot D_{12} + a_8 \cdot D_{13} - a_2 \cdot D_{14} \end{aligned}$$

— — —

$$\begin{aligned} D_{12} &= \Sigma xz \cdot a_6 \cdot a_9 + a_7 \cdot a_5 \cdot \Sigma z + a_1 \cdot \Sigma yz \cdot a_5 - && \text{faktori} \\ &- \Sigma z \cdot a_6 \cdot a_1 - a_5 \cdot a_5 \cdot \Sigma xz - a_9 \cdot \Sigma yz \cdot a_7 = \\ &= \Sigma xz (a_6 \cdot a_9 - a_5 \cdot a_5) + \Sigma z (a_7 \cdot a_5 - a_6 \cdot a_1) + \Sigma yz (a_1 \cdot a_5 - a_9 \cdot a_7) \end{aligned}$$

$$\begin{aligned} D_{13} &= \Sigma xz \cdot a_7 \cdot a_9 + a_2 \cdot a_5 \cdot \Sigma z + a_1 \cdot \Sigma yz \cdot a_1 - && \text{faktori} \\ &- \Sigma z \cdot a_7 \cdot a_1 - a_1 \cdot a_5 \cdot \Sigma xz - a_9 \cdot \Sigma yz \cdot a_2 = \\ &= \Sigma xz (a_7 \cdot a_9 - a_1 \cdot a_5) + \Sigma z (a_2 \cdot a_5 - a_7 \cdot a_1) + \Sigma yz (a_1 \cdot a_1 - a_9 \cdot a_2) \end{aligned}$$

$$\begin{aligned} D_{14} &= \Sigma xz \cdot a_7 \cdot a_5 + a_2 \cdot a_6 \cdot \Sigma z + a_7 \cdot \Sigma yz \cdot a_1 - && \text{faktori} \\ &- \Sigma z \cdot a_7 \cdot a_7 - a_1 \cdot a_6 \cdot \Sigma xz - a_5 \cdot \Sigma yz \cdot a_2 = \\ &= \Sigma xz (a_7 \cdot a_5 - a_1 \cdot a_6) + \Sigma z (a_2 \cdot a_6 - a_7 \cdot a_7) + \Sigma yz (a_7 \cdot a_1 - a_5 \cdot a_2) \end{aligned}$$

-4-

$$\begin{aligned} D_2 &= a_4 \cdot D_{21} - \sum x^2 z D_{22} + a_8 D_{23} - a_2 D_{24} = \\ &= a_4 \cdot D_{12} - \sum x^2 z D_{02} + a_8 D_{23} - a_2 D_{24} \end{aligned}$$

— · —

$$\begin{aligned} D_{23} &= a_3 \cdot \Sigma_{yz} \cdot a_9 + \Sigma_{xz} \cdot a_5 \cdot a_2 + a_1 \cdot a_8 \cdot \Sigma_{z2} - \text{felírni} \\ &\quad - a_2 \cdot \Sigma_{yz} \cdot a_1 - \Sigma_{z2} \cdot a_5 \cdot a_3 - a_9 \cdot a_8 \cdot \Sigma_{xz} = \\ &= \Sigma_{yz} (a_3 \cdot a_9 - a_2 \cdot a_1) + \Sigma_{xz} (a_5 \cdot a_2 - a_9 \cdot a_8) + \Sigma_{z2} (a_1 \cdot a_8 - a_5 \cdot a_3) \end{aligned}$$

$$\begin{aligned} D_{24} &= a_3 \cdot \Sigma_{yz} \cdot a_5 + \Sigma_{xz} \cdot a_6 \cdot a_2 + a_7 \cdot a_8 \cdot \Sigma_{z2} - \text{felírni} \\ &\quad - a_2 \cdot \Sigma_{yz} \cdot a_7 - \Sigma_{z2} \cdot a_6 \cdot a_3 - a_5 \cdot a_8 \cdot \Sigma_{xz} = \\ &= \Sigma_{yz} (a_3 \cdot a_5 - a_2 \cdot a_7) + \Sigma_{xz} (a_6 \cdot a_2 - a_5 \cdot a_8) + \Sigma_{z2} (a_7 \cdot a_8 - a_6 \cdot a_3) \end{aligned}$$

— 5 —

$$D_3 = a_4 \cdot D_{31} - a_3 \cdot D_{32} + \Sigma x^2 z \cdot D_{33} - a_2 \cdot D_{34} =$$

$$= -a_4 D_{13} - a_3 \cdot D_{23} + \Sigma x^2 z \cdot D_{03} - a_2 \cdot D_{34}$$

— . —

$$D_{34} = a_3 \cdot a_7 \cdot \Sigma z + a_2 \Sigma yz \cdot a_2 + \Sigma xz \cdot a_2 \cdot a_1 -$$
$$- a_2 \cdot a_7 \cdot \Sigma xz - a_1 \Sigma yz \cdot a_3 - \Sigma z \cdot a_2 \cdot a_2 =$$

$$= \Sigma z (a_3 \cdot a_7 - a_2 \cdot a_2) + \Sigma yz (a_2 \cdot a_2 - a_1 \cdot a_3) + \Sigma xz (a_2 \cdot a_1 - a_2 \cdot a_7)$$

$$D_4 = a_4 \cdot D_{41} - a_3 \cdot D_{42} + a_2 \cdot D_{43} - \Sigma x^2 z \cdot D_{44} =$$

$$= a_4 \cdot D_{14} + a_3 \cdot D_{24} + a_2 \cdot D_{34} - \Sigma x^2 z \cdot D_{04}$$

$$a = \frac{D_1}{D_0} \quad b = \frac{D_2}{D_0} \quad c = \frac{D_3}{D_0} \quad d = \frac{D_4}{D_0}$$

$$z = ax^2 + bx + cy + d$$

$$19) z = ax^2 + bxy^2 + cx + dy + e$$

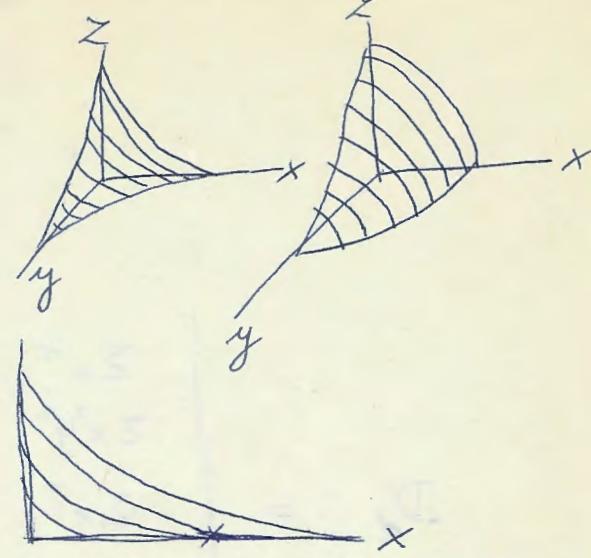
$$z = ax^2 + by^2 + cx + dy + e$$

Elliptikus paraboloid (ha $a \neq 0$ & arányos y előjelű)

Hiperbolikus paraboloid (ha $a \neq 0$ & ellentétes előjelű)

ill. ellipszis sor, vagy hiperbola sor (a nélkülben)

$$a = \frac{D_a}{D}; \quad b = \frac{D_b}{D}; \quad c = \frac{D_c}{D}; \quad d = \frac{D_d}{D}; \quad e = \frac{D_e}{D}$$



$$D = \begin{vmatrix} \sum x^4 & \sum x^2y^2 & \sum x^3 & \sum x^2y & \sum x^2 \\ \sum x^2y^2 & \sum y^4 & \sum xy^2 & \sum y^3 & \sum y^2 \\ \sum x^3 & \sum xy^2 & \sum x^2 & \sum xy & \sum x \\ \sum x^2y & \sum y^3 & \sum xy & \sum y^2 & \sum y \\ \sum x^2 & \sum y^2 & \sum x & \sum y & n \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum zx^2 & \sum x^2y^2 & \sum x^3 & \sum x^2y & \sum x^2 \\ \sum zy^2 & \sum y^4 & \sum xy^2 & \sum y^3 & \sum y^2 \\ \sum zx & \sum xy^2 & \sum x^2 & \sum xy & \sum x \\ \sum zy & \sum y^3 & \sum xy & \sum y^2 & \sum y \\ \sum z & \sum y^2 & \sum x & \sum y & n \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum x^4 & \sum zx^2 & \sum x^3 & \sum x^2y & \sum x^2 \\ \sum x^2y^2 & \sum zy^2 & \sum xy^2 & \sum y^3 & \sum y^2 \\ \sum x^3 & \sum zx & \sum x^2 & \sum xy & \sum x \\ \sum x^2y & \sum zy & \sum xy & \sum y^2 & \sum y \\ \sum x^2 & \sum z & \sum x & \sum y & n \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum x^4 & \sum x^2y^2 & \sum zx^2 & \sum x^2y & \sum x^2 \\ \sum x^2y^2 & \sum y^4 & \sum zy^2 & \sum y^3 & \sum y^2 \\ \sum x^3 & \sum xy^2 & \sum zx & \sum xy & \sum x \\ \sum x^2y & \sum y^3 & \sum zy & \sum y^2 & \sum y \\ \sum x^2 & \sum zy^2 & \sum z & \sum y & n \end{vmatrix}$$

$$D_d = \begin{vmatrix} \sum x^4 & \sum x^2 y^2 & \sum x^3 & \sum z x^2 & \sum x^2 \\ \sum x^2 y^2 & \sum y^4 & \sum x y^2 & \sum z y^2 & \sum y^2 \\ \sum x^3 & \sum x y^2 & \sum x^2 & \sum z x & \sum x \\ \sum x^2 y & \sum y^3 & \sum x y & \sum z y & \sum y \\ \sum x^2 & \sum y^2 & \sum x & \sum z & n \end{vmatrix}$$

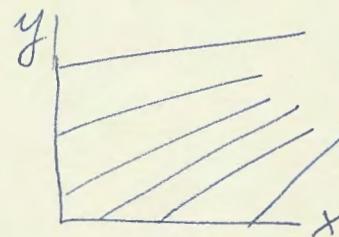
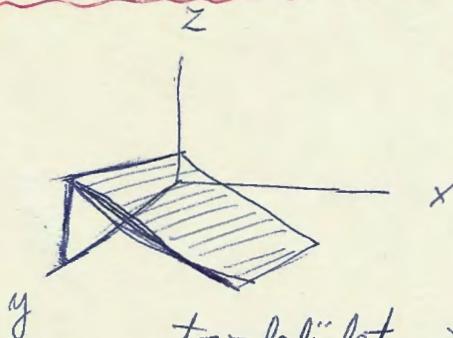
$$D_e = \begin{vmatrix} \sum x^4 & \sum x^2 y^2 & \sum x^3 & \sum x^2 y & \sum z x^2 \\ \sum x^2 y^2 & \sum y^4 & \sum x y^2 & \sum y^3 & \sum z y^2 \\ \sum x^3 & \sum x y^2 & \sum x^2 & \sum x y & \sum z x \\ \sum x^2 y & \sum y^3 & \sum x y & \sum y^2 & \sum z y \\ \sum x^2 & \sum y^2 & \sum x & \sum y & \sum z \end{vmatrix}$$

Meghatározandó: $\sum x^4 \quad \sum y^4 \quad \sum x^2 y^2$
 $\sum x^3 \quad \sum y^3 \quad \sum x^2 y \quad \sum x y^2 \quad \sum z x^2 \quad \sum z y^2$
 $\sum x^2 \quad \sum y^2 \quad \sum x y \quad \sum z x \quad \sum z y$
 $\sum x \quad \sum y \quad \sum z$

Leveretés: Bánya komohok és körök nemelőnlásának analitikus függvénye

20) $z = (axx + bxy + c)/(x+1)$

$$M = \frac{az + by + c}{x+1} = a \frac{x}{x+1} + b \frac{y}{x+1} + c \frac{1}{x+1}$$



nem egy pont-
ban met-
nődnek

továbbfelület, ill. nem párhuzamos futású egyenesek (a né-
gan)

$$a = \frac{D_a}{D}; \quad b = \frac{D_b}{D}; \quad c = \frac{D_c}{D}$$

levezetés: Bolyai-konjunktíumok és
hármasok nemelőnlásá-
nak analitikus függ-
vényje

és 1973 előtt Bernárd-féle
utpaligárdanépi téma
munkája

$$D = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \frac{xy}{(x+1)^2} & \sum \frac{x}{(x+1)^2} \\ \sum \frac{xy}{(x+1)^2} & \sum \frac{y^2}{(x+1)^2} & \sum \frac{y}{(x+1)^2} \\ \sum \frac{x}{(x+1)^2} & \sum \frac{y}{(x+1)^2} & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum \frac{xz}{x+1} & \sum \frac{xy}{(x+1)^2} & \sum \frac{x}{(x+1)^2} \\ \sum \frac{yz}{x+1} & \sum \frac{y^2}{(x+1)^2} & \sum \frac{y}{(x+1)^2} \\ \sum \frac{z}{x+1} & \sum \frac{y}{(x+1)^2} & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

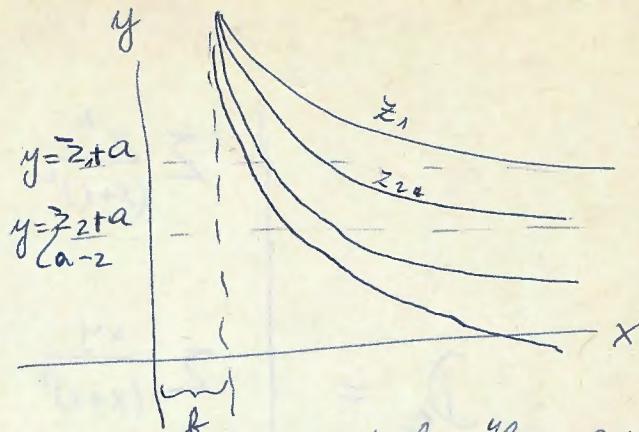
$$D_b = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \frac{xz}{x+1} & \sum \frac{x}{(x+1)^2} \\ \sum \frac{xy}{(x+1)^2} & \sum \frac{yz}{x+1} & \sum \frac{y}{(x+1)^2} \\ \sum \frac{x}{(x+1)^2} & \sum \frac{z}{x+1} & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \frac{xy}{(x+1)^2} & \sum \frac{xz}{x+1} \\ \sum \frac{xy}{(x+1)^2} & \sum \frac{y^2}{(x+1)^2} & \sum \frac{yz}{x+1} \\ \sum \frac{x}{(x+1)^2} & \sum \frac{y}{(x+1)^2} & \sum \frac{z}{x+1} \end{vmatrix}$$

$$21) z = (axx + bxy - xxy + c)/(x+1)$$

$$z = \frac{ax + by - xy + c}{x+1}$$

A gyakorlatban még nem próbáltam ki, levezetésre az útbeton témaúl (1973-75) van, de nem alkalmaztam. A konvergencia függvényt lehetne így felírni, de nem volt rá időm.



hiperbola sor, amelynek függőleges asymptotája konstans ($x=8$), másik rész aximatajára változó (mindegyik más és más

$$a = \frac{Da}{D}; \quad b = \frac{Db}{D}; \quad c = \frac{Dc}{D}$$

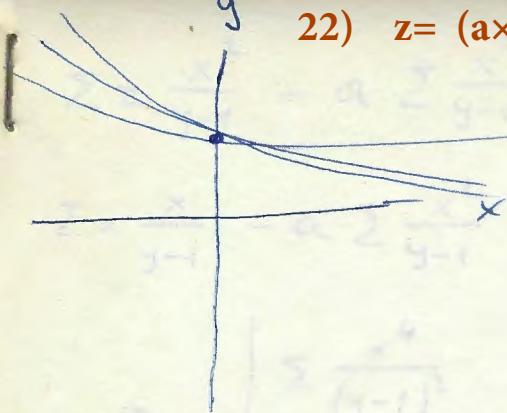
$$D = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \frac{xy}{(x+1)^2} & \sum \frac{x}{(x+1)^2} \\ \sum \frac{xy}{(x+1)^2} & \sum \frac{y^2}{(x+1)^2} & \sum \frac{y}{(x+1)^2} \\ \sum \frac{x}{(x+1)^2} & \sum \frac{y}{(x+1)^2} & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum \left(\frac{zx}{x+1} + \frac{x^2y}{(x+1)^2} \right) & \sum \frac{xy}{(x+1)^2} & \sum \frac{x}{(x+1)^2} \\ \sum \left(\frac{zy}{x+1} + \frac{xy^2}{(x+1)^2} \right) & \sum \frac{y^2}{(x+1)^2} & \sum \frac{y}{(x+1)^2} \\ \sum \left(\frac{z}{x+1} + \frac{xy}{(x+1)^2} \right) & \sum \frac{y}{(x+1)^2} & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \left(\frac{zx}{x+1} + \frac{x^2y}{(x+1)^2} \right) & \sum \frac{x}{(x+1)^2} \\ \sum \frac{xy}{(x+1)^2} & \sum \left(\frac{zy}{x+1} + \frac{xy^2}{(x+1)^2} \right) & \sum \frac{y}{(x+1)^2} \\ \sum \frac{x}{(x+1)^2} & \sum \left(\frac{z}{x+1} + \frac{xy}{(x+1)^2} \right) & \sum \frac{1}{(x+1)^2} \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum \frac{x^2}{(x+1)^2} & \sum \frac{xy}{(x+1)^2} & \sum \left(\frac{zx}{x+1} + \frac{x^2y}{(x+1)^2} \right) \\ \sum \frac{xy}{(x+1)^2} & \sum \frac{y^2}{(x+1)^2} & \sum \left(\frac{zy}{x+1} + \frac{xy^2}{(x+1)^2} \right) \\ \sum \frac{x}{(x+1)^2} & \sum \frac{y}{(x+1)^2} & \sum \left(\frac{z}{x+1} + \frac{xy}{(x+1)^2} \right) \end{vmatrix}$$

$$22) z = \frac{ax^2 + bx + c}{y-1}$$



úthányt termelő

$$y = ax^2 + bx + c \quad \text{ha } x=0, \text{ akkor } y=1, \text{ amiből } c=1$$

$$P(0,1) \text{ ponton átmegő parabola: } y = ax^2 + bx + 1$$

Leppener a parabolát hosszúságban:

$$y' = 2ax + b = 0 \quad x = -\frac{b}{2a} = A$$

$$y = \frac{a}{2}x^2 + \frac{b}{2}x + 1$$

$$y - 1 = \frac{1}{2}(ax^2 + bx)$$

$$z = \frac{ax^2 + bx}{y-1} = a \frac{x^2}{y-1} + b \frac{x}{y-1}$$

$$F = \sum \left(z - a \frac{x^2}{y-1} - b \frac{x}{y-1} \right)^2 = \min$$

$$\frac{\partial F}{\partial a} = -2 \sum \left(z - a \frac{x^2}{y-1} - b \frac{x}{y-1} \right) \frac{x^2}{y-1} = 0$$

$$\frac{\partial F}{\partial b} = -2 \sum \left(z - a \frac{x^2}{y-1} - b \frac{x}{y-1} \right) \frac{x}{y-1} = 0$$

$$\sum \frac{x^2}{y-1} - a \sum \frac{x^2}{y-1} \cdot \frac{x^2}{y-1} - b \sum \frac{x}{y-1} \cdot \frac{x^2}{y-1} = 0$$

$$\sum \frac{x}{y-1} - a \sum \frac{x^2}{y-1} \cdot \frac{x}{y-1} - b \sum \frac{x}{y-1} \cdot \frac{x}{y-1} = 0$$

$$D = \begin{vmatrix} \sum \frac{x^4}{(y-1)^2} & \sum \frac{x^3}{(y-1)^2} \\ \sum \frac{x^3}{(y-1)^2} & \sum \frac{x^2}{(y-1)^2} \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum \frac{x^2}{y-1} & \sum \frac{x^3}{(y-1)^2} \\ \sum \frac{x}{y-1} & \sum \frac{x^2}{(y-1)^2} \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum \frac{x^4}{(y-1)^2} & \sum \frac{x^2}{y-1} \\ \sum \frac{x^3}{(y-1)^2} & \sum \frac{x}{y-1} \end{vmatrix}$$

Ki hell mainhami:

$$\frac{x^2}{(y-1)^2} \quad \frac{x^3}{(y-1)^2} \quad \frac{x^4}{(y-1)^2} \quad = \frac{x^2}{y-1} \quad = \frac{x}{y-1}$$

$$y = \frac{a}{z-c} x^2 + \frac{b}{z-c} x + 1$$

-1 -

untuk tampilan

$$z-c = \frac{a}{y-1} x^2 + \frac{b}{y-1} x$$

$$23) z = (axx^2)/(y-1) + (bx)/(y-1) + c$$

$$z = \frac{a}{y-1} x^2 + \frac{b}{y-1} x + c$$

$$F = \sum \left(z - \frac{a}{y-1} x^2 - \frac{b}{y-1} x - c \right)^2 = \min$$

$$\frac{\partial F}{\partial a} = -2 \sum \left(z - \frac{a}{y-1} x^2 - \frac{b}{y-1} x - c \right) \frac{x^2}{y-1} = 0$$

$$\frac{\partial F}{\partial b} = -2 \sum \left(z - \frac{a}{y-1} x^2 - \frac{b}{y-1} x - c \right) \frac{x}{y-1} = 0$$

$$\frac{\partial F}{\partial c} = -2 \sum \left(z - \frac{a}{y-1} x^2 - \frac{b}{y-1} x - c \right) = 0$$

$$\sum z \frac{x^2}{y-1} - a \sum \frac{x^2}{y-1} \cdot \frac{x^2}{y-1} - b \sum \frac{x}{y-1} \cdot \frac{x^2}{y-1} - c \sum \frac{x^2}{y-1} = 0$$

$$\sum z \frac{x}{y-1} - a \sum \frac{x^2}{y-1} \cdot \frac{x}{y-1} - b \sum \frac{x}{y-1} \cdot \frac{x}{y-1} - c \sum \frac{x}{y-1} = 0$$

$$\sum z - a \sum \frac{x^2}{y-1} - b \sum \frac{x}{y-1} - n.c = 0$$

$$D = \begin{vmatrix} \sum \frac{x^4}{(y-1)^2} & \sum \frac{x^3}{(y-1)^2} & \sum \frac{x^2}{y-1} \\ \sum \frac{x^3}{(y-1)^2} & \sum \frac{x^2}{(y-1)^2} & \sum \frac{x}{y-1} \\ \sum \frac{x^2}{y-1} & \sum \frac{x}{y-1} & n \end{vmatrix}$$

$$D_a = \begin{vmatrix} \sum z \frac{x^2}{y-1} & \sum \frac{x^3}{(y-1)^2} & \sum \frac{x^2}{y-1} \\ \sum z \frac{x}{y-1} & \sum \frac{x^2}{(y-1)^2} & \sum \frac{x}{y-1} \\ \sum z & \sum \frac{x}{y-1} & n \end{vmatrix}$$

$$D_b = \begin{vmatrix} \sum \frac{x^4}{(y-1)^2} & \sum \frac{x^2}{y-1} & \sum \frac{x^2}{y-1} \\ \sum \frac{x^3}{(y-1)^2} & \sum z \frac{x}{y-1} & \sum \frac{x}{y-1} \\ \sum \frac{x^2}{y-1} & \sum z & n \end{vmatrix}$$

$$D_c = \begin{vmatrix} \sum \frac{x^4}{(y-1)^2} & \sum \frac{x^3}{(y-1)^2} & \sum z \frac{x^2}{y-1} \\ \sum \frac{x^3}{(y-1)^2} & \sum \frac{x^2}{(y-1)^2} & \sum z \frac{x}{y-1} \\ \sum \frac{x^2}{y-1} & \sum \frac{x}{y-1} & \sum z \end{vmatrix}$$

$$z = a \underbrace{\frac{(1-x^2)}{y}}_u + b \underbrace{\frac{(1-x)}{y}}_v + c = aU + bV + c$$

$$F = \sum (z - aU - bV - c)^2 = \text{min}$$

$$\frac{\partial F}{\partial a} = -2 \sum (z - aU - bV - c)U = 0$$

$$\frac{\partial F}{\partial b} = -2 \sum (z - aU - bV - c)V = 0$$

$$\frac{\partial F}{\partial c} = -2 \sum (z - aU - bV - c) = 0$$

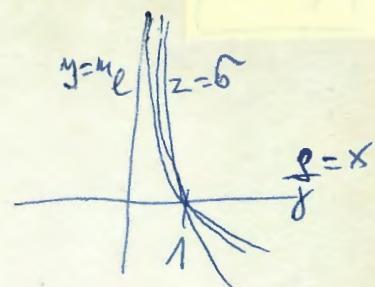
$$\sum zU - a \sum U^2 - b \sum UV - c \sum U = 0$$

$$\sum zV - a \sum UV - b \sum V^2 - c \sum V = 0$$

$$\sum z - a \sum U - b \sum V - cn = 0$$

Konkavitánsó

$$\begin{array}{l} \Sigma U^2 \\ \Sigma U \\ \Sigma UV \\ \Sigma V^2 \\ \Sigma V \\ \Sigma zU \\ \Sigma zV \\ \Sigma z \\ n \end{array}$$



$$D = \begin{vmatrix} \Sigma U^2 & \Sigma UV & \Sigma U \\ \Sigma UV & \Sigma V^2 & \Sigma V \\ \Sigma U & \Sigma V & n \end{vmatrix}$$

$$D_a = \begin{vmatrix} \Sigma zU & \Sigma zV & \Sigma z \\ \Sigma zV & \Sigma V^2 & \Sigma V \\ \Sigma z & \Sigma V & n \end{vmatrix}$$

$$24) z = (ax(1-x^2))/y + (bx(1-x))/y + c$$

$$D_b = \begin{vmatrix} \Sigma U^2 & \Sigma zU & \Sigma U \\ \Sigma UV & \Sigma zV & \Sigma V \\ \Sigma U & \Sigma z & n \end{vmatrix}$$

$$D_c = \begin{vmatrix} \Sigma U^2 & \Sigma UV & \Sigma zU \\ \Sigma UV & \Sigma V^2 & \Sigma zV \\ \Sigma U & \Sigma V & \Sigma z \end{vmatrix}$$

$$25) z = x/(ax \ln y) + a/b$$

Programjáték
kidolgozta

46 pp.

$$y = e^{\frac{x}{az - b}}$$

$$z = \frac{x}{a \ln y} + \frac{b}{a}$$

$$z = A \frac{x}{\ln y} + B$$

$$\text{ha } A = \frac{1}{a} \quad B = \frac{b}{a}$$

$$a = \frac{1}{A} \quad b = \frac{B}{A}$$

mert $\ln y = \frac{x}{az - b}$

$$az - b = \frac{x}{\ln y}$$

$$az = \frac{x}{\ln y} + b$$

$$z = \frac{x}{a \ln y} + \frac{b}{a}$$

A függvény tulajdonságai:

$$x=0 \text{ esetén } y=1$$

$$z=\infty \text{ esetben } y=1$$

$$z=0 \text{ esetben } y=e^{-\frac{x}{b}}$$

$$F = \sum (z - A \frac{x}{\ln y} - B)^2 = \min \quad \text{legyen } u = \frac{x}{\ln y}$$

$$\frac{\partial F}{\partial A} = -2 \sum (z - Au - B)u = 0$$

$$\frac{\partial F}{\partial B} = -2 \sum (z - Au - B) = 0$$

$$\sum zu - \sum Au^2 - \sum Bu = 0$$

$$\sum z - \sum Au - \sum B = 0$$

$$\sum zu - A \sum u^2 - B \sum u = 0$$

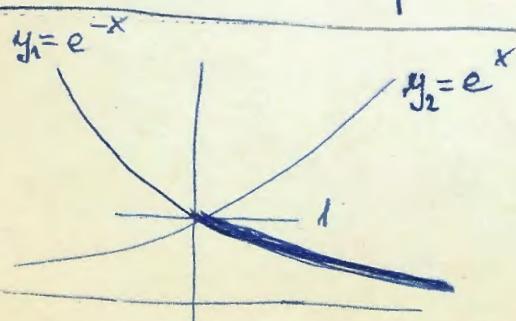
$$\sum z - A \sum u - nB = 0$$

$$D = \begin{vmatrix} \sum u^2 & \sum u \\ \sum u & n \end{vmatrix}$$

$$D_A = \begin{vmatrix} \sum zu & \sum u \\ \sum z & n \end{vmatrix}$$

$$A = \frac{D_A}{D} \quad B = \frac{D_B}{D}$$

$$D_B = \begin{vmatrix} \sum u^2 & \sum zu \\ \sum u & \sum z \end{vmatrix}$$



Mint a vantarion kihurrott rör érdekel, tehát $az - b < 0$ kell legyen, ill. a függvény normál esetében a negatív hármasoknak, ahol $az < b$ $z > \frac{b}{a} = B$ (mert "a" negatív ndm). Az $az - b = 0$, ill. $z = \frac{b}{a} = B$ helyen van az y_1 ar y_2 -be, ha $z > B$, akkor $z - B > 0$, araz $A < 0$ kell legyen, mert $\frac{x}{\ln y}$ esetünkben negatív nán (alakában)

$$y = e$$

$$(az + b)x$$

$$26) z = (\ln y \pm bx)/ax$$

amiböl

Az $y = 1$ ponton

$$z = \frac{\ln y \pm bx}{ax} = \frac{\ln y}{ax} \pm \frac{b}{a} = A \frac{\ln y}{x} + B$$

$$\text{tua } A = \frac{1}{a} \quad \text{és } B = \frac{b}{a}$$

$$\text{mert } \ln y = (az + b)x \quad azx = \ln y \pm bx$$

atmenő x körül
asymptotikus
exponentialis
függvény az

$$F = \sum \left(z_i - A \frac{\ln y}{x} \mp B \right)^2 = \min$$

$$\frac{\partial F}{\partial A} = -2 \sum \left(z_i - A \frac{\ln y}{x} \mp B \right) \left(\pm \frac{\ln y}{x} \right) = 0$$

$$\frac{\partial F}{\partial B} = -2 \sum \left(z_i - A \frac{\ln y}{x} \mp B \right) = 0$$

$$\sum z \left(\pm \frac{\ln y}{x} \right) \mp \sum A \frac{\ln y}{x} \cdot \frac{\ln y}{x} - \sum B \frac{\ln y}{x} = 0$$

$$\sum z - \sum A \frac{\ln y}{x} \mp \sum B = 0$$

$$\frac{\ln y}{x} = u$$

$$+ \sum z u \mp \sum A u^2 - \sum B u = 0$$

$$\sum z - \sum A u \mp \sum B = 0$$

$$+ \sum z u \mp A \sum u^2 - B \sum u = 0$$

$$+ \sum z = A \sum u + n \cdot B = 0$$

$$D = \begin{vmatrix} \sum u^2 & \sum u \\ \sum u & n \end{vmatrix}$$

$$D_A = \begin{vmatrix} \sum z u & \sum u \\ \sum z & n \end{vmatrix}$$

$$D_B = \begin{vmatrix} \sum u^2 & \sum z u \\ \sum u & \sum z \end{vmatrix}$$

$$A = \frac{D_A}{D} \quad B = \frac{D_B}{D}$$

$$a = \frac{1}{A} \quad b = \frac{B}{A}$$

az a fr. enar ott hennálható, ahol $az - b < 0$, enar $z = \frac{b}{a} = B$ (mert az "a" pozitív szám)

27) Korrelációs jellemzők

KORRELÁCIÓS JELLEMZÖK

Változás mellettégi változás
szorosságát illetően

Kovariancia.
(három türelemben)

$$\text{cov} = \frac{\sum (x - \bar{x})(y - \bar{y})}{n-1}$$

[x, y terület]

Korrelációs koeficialus.
(három türelemben)

$$r = \frac{\text{cov}}{\sqrt{\frac{1}{n-1} \sum (x - \bar{x})^2 \cdot \sum (y - \bar{y})^2}}$$

nincs
dimenziója

Korrelációs index.
(három türelemben)

$$\gamma = \sqrt{1 - \frac{F}{\sum (y - \bar{y})^2}}$$

nincs dimenziója

$$F = \sum (y - \bar{y})^2$$

\bar{y} = mért érték

\bar{y} = számított érték

"standard liba"
(három türelemben)

$$s = \sqrt{\frac{F}{n}}$$

[mint y]

Relatív liba
(hét türelemben)

$$H = 100 \cdot \frac{s}{\frac{\sum y}{n}}$$

[%]

Haromnálterős corr. index

f

R = Totális v. teljes korrelációs index (regr. próbée')

$$R = \sqrt{1 - \frac{\sum (z_i - \bar{z})^2}{\sum (z_i - \bar{z})^2}}$$

z_i = mérő

\bar{z} = ordináltott

\bar{z} = a mérő értékek átlaga

$$\text{Standard liba} = s_z = \sqrt{\frac{\sum (z - \bar{z})^2}{n}}$$

$$\text{Relatív liba} H = \frac{s}{\bar{z}}$$

r = totális corr. egyséthez (támasztalati)

$$r = \frac{\sqrt{n} \sum (x_i - \bar{x})(y_i - \bar{y})(z_i - \bar{z})}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2 \cdot \sum (z_i - \bar{z})^2}}$$

Csak lineáris reprezentáció
igaz

$$S = \frac{M \left\{ [x - M(x)][y - M(y)][z - M(z)] \right\}}{D(x) \cdot D(y) \cdot D(z)}$$

$$\begin{array}{l} M(x) = \bar{x} = \frac{\sum x_i}{n} \\ M(y) = \bar{y} = \frac{\sum y_i}{n} \\ M(z) = \bar{z} = \frac{\sum z_i}{n} \end{array} \quad \left| \begin{array}{l} x - M(x) = x_i - \bar{x} \\ y - M(y) = y_i - \bar{y} \\ z - M(z) = z_i - \bar{z} \end{array} \right.$$

$$M \left\{ [x - M(x)][y - M(y)][z - M(z)] \right\}$$

$$= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})(z_i - \bar{z})}{n}$$

$$D^2(x) = M \left\{ [x - M(x)]^2 \right\} = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$D^2(y) = \frac{\sum (y_i - \bar{y})^2}{n} \qquad D^2(z) = \frac{\sum (z_i - \bar{z})^2}{n}$$

$$\begin{aligned}
 & \frac{\sum (x_i - \bar{x})(y_i - \bar{y})(z_i - \bar{z})}{n} \\
 p &= \frac{\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \cdot \sqrt{\frac{\sum (y_i - \bar{y})^2}{n}} \cdot \sqrt{\frac{\sum (z_i - \bar{z})^2}{n}}}{\frac{\sum (x_i - \bar{x})(y_i - \bar{y})(z_i - \bar{z})}{n}} = \\
 &= \frac{\frac{s_x}{n} \cdot \frac{N}{n \cdot \sqrt{n}} = \frac{s_x \cdot \sqrt{n} \cdot s_z}{n \cdot N}}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2 \cdot \sum (z_i - \bar{z})^2}} = \\
 &= \frac{\underbrace{\sqrt{n} \cdot \sqrt{n} \cdot \sqrt{n}}_n \cdot \frac{\sum (x_i - \bar{x})(y_i - \bar{y})(z_i - \bar{z})}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2 \cdot \sum (z_i - \bar{z})^2}} \\
 &= \text{melan } p = 2
 \end{aligned}$$

Parciális

-13 -

$$r_{zy|x} = \frac{r_{zx} - r_{zy} \cdot r_{xy}}{\sqrt{(1 - r_{zy}^2)(1 - r_{xy}^2)}} \quad \text{tökéletes}$$

$$r_{zy|x} = \frac{r_{zy} - r_{zx} \cdot r_{xy}}{\sqrt{(1 - r_{zy}^2) \cdot (1 - r_{xy}^2)}} \quad *$$

$$r_{zx} = \frac{\sum (z_i - \bar{z})(x_i - \bar{x})}{\sqrt{\sum (z_i - \bar{z})^2 \sum (x_i - \bar{x})^2}}$$

$$r_{zy} = \frac{\sum (z_i - \bar{z})(y_i - \bar{y})}{\sqrt{\sum (z_i - \bar{z})^2 \cdot \sum (y_i - \bar{y})^2}}$$

$$r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2}}$$

Elnézést a kézírásért..